A Quick and Dirty Guide to Critical Thinking

by

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Chapter 1: Introduction to Critical Thinking

1. Basic Concepts

Critical thinking involves using the techniques of logic in order to help determine whether or not we ought to believe the various things we read about or people tell us. This, of course, raises the question as to what logic is. **Logic** is the discipline that evaluates arguments, that is, that tells us whether they are good or bad. As such, logic consists of various methods for evaluating arguments we encounter, which can also be used to construct good arguments of our own.

Of course, this raises another question: exactly what is an argument? The first thing to note is, that in logic at least, an argument is not a mere verbal fight of the kind that children sometimes have with their parents. Rather an **argument** consists of a group of statements or propositions, one or more of which are claimed to provide reasons or evidence for one (or more) of the others. The statements/ propositions that present reasons or evidence are the **premises** of the argument; and the statement/ proposition that the evidence is claimed to support is the **conclusion**. The idea is that an argument is a group or collection of statements that has a certain kind of structure. Consider the following passage:

> Snow is white. A Toyota Corolla is a small sedan. Saskatchewan is a province in the Canadian Prairies.

It consists of a collection of statements, but it is unstructured. None of the statements has any kind of special status within the passage. And none of the statements are designed to elaborate on, or illustrate, or explain, or provide reasons or evidence for any of the others. Consider now the following:

> Cheating requires deliberately breaking the rules of a game but computers lack free will. It follows that computers cannot cheat.

Notice that in this case one of the statements – "Computers cannot cheat" – has a special status: the point of the passage is to get the reader to believe it. Moreover, the other statements – "Cheating requires deliberately breaking the rules of a game" and "computers lack free will" – are offered as reasons for thinking this special statement is true. As result, although the first passage lacks an argument, the second passage contains an argument. Note: although there are differences between them, we will be using the terms "statement" and "proposition" more or less interchangeably.

Finally, it is worth noting that not every sentence is a statement and, hence, not every collection of sentences is a collection of statements. A **statement** is a sentence that is used to make a claim about how things are and, as such, is (normally) either true or false: if things are as they are claimed to be then the statement is true; if things are not as they are claimed to be then the statement is false. Consider the following sentences:

> There are 60 seconds in a minute
> Coupes have two doors and sedans have four doors
> Dogs are felines
> Saint John is the capital city of New Brunswick

The first sentence makes a claim about the number of seconds in a minute and since, as a matter of fact, there are 60 seconds in a minute, this statement is true. Similarly, since the second sentence makes a correct claim about the number of doors found in various styles of cars, it too is a true statement. The third and fourth statements make claims about the familial categorization of dogs and the political geography of New Brunswick respectively and, as a result, both count as statements. But since the claims they make are inaccurate, they count as false statements. Note: the falsity of a statement does not prevent it from serving as a premise or conclusion of an argument.

It is important to note, however, that not all sentences count as statements. Making claims about how things are is just one of the things people use language to do. Other things people do with language include asking questions, making promises, issuing commands, and the like. And when sentences are used to these other sorts of speech acts, it does not make sense to attribute truth or falsity to them. Consider, for example, the following:

> What is the capital city of New Brunswick?
> I promise to pay you back tomorrow.
> Take your shoes of before you come into the house.

Since it does not make a whole lot of sense to suppose that they are (or even could be) true or false, these sentences do not count as statements. Rather they are better understood respectively as a question, a promise, and a command. Note: the type of speech act performed by means of uttering a sentence depends in large part of the intentions of the speaker – what kind of impact/ response she or he was trying to get from his or her audience; but given that, for example, declarative sentences are conventionally used to make statements, we will not require independent evidence of speakers' intentions to make such determinations.

Exercises Section 1.1

Indicate whether or not each of the following sentences is a statement. If it is, determine whether it is true or false; if it is not, determine what kind of speech act it is (question, command, request, promise, etc.).

1. Please pass the collard greens.

2. The Pacific is the largest ocean on the planet.

3. Get upstairs and clean your room

4. All vaccines are completely ineffective in the prevention of disease.

5. Where did you leave the car keys?

6. The chemical composition of table salt is NaCl.

7. I bet you five dollars the Roughriders will beat the Argonauts.

8. Canada is the most populous nation in the world.

9. I promise I will pick you up after the game.

10. What is the air speed velocity of an unencumbered swallow?

11. Would you pick up the laundry on the way home, please?

12. If Mary has a dog as a pet, then she has a mammal as a pet.

13. Bilbo Baggins cheated in the riddle game he played with Gollum.

14. You need to finish your homework before you play any computer games

15. The present king of France is bald.

2. Types of Arguments

A central goal of critical thinking is to evaluate arguments as good or bad, that is, whether or not they provide with adequate grounds for accepting their conclusions. The standards we use to evaluate arguments – and the techniques we use to determine if those standards are met – vary, however, with the type of argument we are considering. As a result, before we can assess arguments, we need to be able to determine what sorts of arguments we are dealing with. One central distinction is between deductive and inductive arguments. These types of arguments differ in the claims made about the nature of the inferential relationships that hold between the premises and the conclusion. Someone who makes a **deductive argument** is claiming that it is impossible for the conclusion to be false given that the premises are true. In effect, the arguer is claiming that the truth of the premises guarantees the truth of the conclusion: since the premises are true, the conclusion must be true as well. It is worth noting that the premises of a deductive argument might not, in fact, guarantee the truth of the conclusion. But what makes an argument deductive is the claim that the premises guarantee the truth of the conclusion, whether or not this is in fact so.

Someone who makes an **inductive argument**, in contrast, is claiming that it is improbable that the conclusion is false given that the premises are true. This is equivalent to claiming that the truth of the premises makes the conclusion likely: since the premises are true, the conclusion is probably true. As above, the premises of an inductive argument might not make the conclusion likely; what makes an argument inductive is the claim that this is so, not the reality.

Whether an argument counts as deductive or inductive depends on the nature of the inferential relationship that is claimed by the arguer to hold between the premises and the conclusion. Two central sources of evidence of the kind of inferential claim that is made include the following sorts of features of the argument at issue: (i) the presence of certain indicator terminology and (ii) the actual strength of the inferential connection. Note: in some cases, there may simply be insufficient evidence to make this determination.

First, indicator terminology for distinguishing between inductive and deductive arguments includes the following:

> *Deductive*: necessarily, certainly, absolutely, definitely
>
> *Inductive*: probable, improbable, plausible, implausible, likely, unlikely

Consider the following argument.

> Francis McGee is a professor, and professors are extreme liberals. Therefore, *necessarily* Francis McGee is an extreme liberal.

The presence of the term “necessarily” indicates that this is a deductive argument. But if the argument was rewritten with different indicator terminology, such as

> Francis McGee is a professor, and professors are extreme liberals. Therefore, *probably* Francis McGee is an extreme liberal

this would count instead as evidence that it is inductive.

Second, the actual strength of the inferential connection between the premises and the conclusion can count as evidence for what kind of inferential claim has been made. If, for example, the conclusion of an argument *in fact* follows with strict necessity from the premises, then it is reasonable to assume that the arguer is *claiming* that the conclusion follows with strict necessity from the premises rather than merely that the premises make the conclusion probable. As a result, one could conclude on this basis that the argument is deductive. But if instead the conclusion only *in fact* follows probably from the premises, then it is reasonable to assume that the arguer is *claiming* only that the conclusion follows probably from the premises. Hence, this would count as evidence that the argument is inductive. Consider the following argument:

> All dogs are mammals, and all mammals are animals. Hence, all dogs are animals.

Since it is *in fact* impossible for the conclusion to be false given the truth of the premises, this counts as evidence that the argument is deductive. Now consider the following argument.

> Most politicians are partisan hacks. So, since Justin Trudeau is a politician, he is a partisan hack.

Since the truth of the premises *in fact* makes the truth of the conclusion likely, without guaranteeing it, this counts as evidence that the argument is inductive.

Exercises Section 1.2

Determine whether each of the following arguments is inductive or deductive.

1. All docile idiots are doomed to fail. Ichabod Crane is a docile idiot. Hence, Ichabod Crane is definitely doomed to fail.

2. At least one U of S professor is an accomplished harpist. Professor Moore teaches at U of S. It follows that Professor Moore is probably an accomplished harpist.

3. Either Fred will pay what he owes her or Mary will slash his tires. Fred will not pay what he owes her. Hence, Mary will slash Fred's tires.

4. If it is snowing, then it is cold. It is cold. It follows that it is certainly snowing.

5. Most pets are either cats or dogs. Rashid's pet, Fido, is not a cat. Hence, Fido is a dog.

6. It is usually cold out when it is snowing. As a result, since it is currently snowing, it follows that it is likely cold out.

7. All dogs are good pets. Some dogs are poorly trained. Therefore, some good pets are poorly trained.

8. Candice forgot her favorite cinderblock out behind the 7-Eleven. Cinderblocks are usually just left where they are in low traffic areas. Hence, since almost no one goes out behind the 7-Eleven, it follows that Candice's cinderblock is still there.

9. If Albert learns how to hoot like an owl, then either Jane will lock him out of the house or Myrna will leave. Jane will not lock Albert out. Hence, necessarily, if Albert learns how to hoot like an owl, then Myrna will leave.

10. All people with poor personal hygiene are unpopular dinner guests. No unpopular dinner guests are invited to tea with the Queen. Hence, no people with poor personal hygiene are invited to tea with the Queen.

11. The last time I ate Tanya's 5-alarm chili it upset my stomach. And when my stomach gets upset I sometimes "cut the cheese." As a result, you should probably prepare yourself for a malodorous experience.

12. No logic quizzes are recreational activities. Some recreational activities don't require practicing. It follows that some logic quizzes definitely don't require practicing.

13. Marc will either do his laundry or clean his bathroom. If Marc does his laundry, then he will impress his father. And if Marc cleans his bathroom, then he will impress his mother. Therefore, it is certain that Marc will impress either his mother or his father.

14. Every night this week I was woken up by the sound of a car door being slammed shut. And I can never get back to sleep when I wake up in the middle of the night. Hence, it is likely that I will never have a good night's sleep ever again.

15. Jenna almost always holds a grudge when she feels people are disrespectful towards her. And she considers the failure to say "please" and "thank you" to be disrespectful. As a result, since you didn't thank her for driving you home, she is probably going to hold a grudge.

3. Evaluating Arguments

Now that we can distinguish between inductive and deductive arguments, we are in a position to introduce the standards for evaluating these kinds of arguments. To evaluate an argument is to determine whether it is good or bad. A good argument is one in which the premises provide adequate reasons or evidence for the conclusion, such that it is reasonable to believe the conclusion on that basis. And what counts as adequate reasons or evidence depends on whether the argument at issue is inductive or deductive. It is worth noting that in subsequent chapters we will be developing techniques designed to help us determine whether or not the relevant standards have been met.

A. Evaluating Deductive Arguments

A good deductive argument has to have two features. First, it needs to be valid. A deductive argument is **valid** just in case it is not possible for the premises to be true and the conclusion false. If it is possible for the premises to be true and the conclusion false, then the argument is **invalid**. In effect, validity consists in the truth of the inferential claim made in deductive arguments. Consider the following argument.

> All corgis are dogs, and all dogs are mammals. It follows that all corgis are mammals.

Since the truth of the premises guarantees the truth of the conclusion, the argument is valid. Now consider the following.

> All lunch items are meals, and all sandwich platters are meals. It follows that all lunch items are sandwich platters.

Even if both premises are true, there might still be lunch items that are not sandwiches, that is, it would still be possible for the conclusion to be false. As a result, since the truth of the premises does not guarantee the truth of the conclusion, this argument is invalid.

What is important to note, however, is that validity by itself is not sufficient for a good deductive argument. Consider the following.

> All pigs have wings, and all things with wings can fly. It follows that all pigs can fly.

This is a bad argument, and on its basis we have no reason to believe that all (or any) pigs can fly. But it is valid. After all, if both premises were true, the conclusion would have to be true as well. The trouble is that both of the premises are false. As a result, even though the argument is valid, it gives us no reason to believe that the conclusion is true. What is required for a deductive argument to be good is for it to be sound. And an argument is **sound** just in case (i) it is valid and (ii) it has all true premises. If an argument is valid, you know that if the premises are true, then the conclusion will be true as well. So, since the premises in a sound argument are in fact true, the conclusion is guaranteed to be true.

B. Evaluating Inductive Arguments

A good inductive argument has two features as well. First, it needs to be strong. In a **strong** inductive argument it is improbable that the conclusion is false given that the premises are true. In a **weak** inductive argument, by contrast, the conclusion does not follow probably from the premises. Consider the following argument.

> Most dogs are good around children. So, since Fido is a dog, Fido is good around children.

Since the conclusion is likely assuming the truth of the premises, this argument is strong. Now consider

> A few guitars in Saskatoon crack as a result of the dry conditions. So since Mary's guitar is in Saskatoon, it will crack as a result of the dry conditions.

Given that this conclusion is unlikely given the premises, it follows that this argument is weak. It is worth noting that unlike validity – which is all or nothing – inductive strength comes in degrees. Inductive arguments can be more or less strong depending on how likely the premises make the conclusion.

As with validity, strength by itself is not sufficient for a good inductive argument. Consider the following.

> Most politicians need to drink human blood to survive. So, since Andrew Scheer is a politician, he needs to drink human blood to survive.

This is a bad argument and we should not believe the conclusion on its basis. But it is strong. After all, if the premises were true, the conclusion would probably be true as well. But the first premise is false. As a result, even though the argument is strong, it gives us no reason to believe that the conclusion is true. What is required for an inductive argument to be good is for it to be cogent. And an argument is **cogent** just in case (i) it is strong and (ii) it has all true premises. If an argument is strong, you know that if the premises are true, then the conclusion will probably be true as well. So, since the premises in a cogent argument are in fact true, the conclusion is in fact probably true.

4. Identifying Arguments

A. Arguments

As we have seen the point of a sentence can be to make a statement; but it might instead be to ask a question, issue a command, or make a proposal among other things. Similarly, although the point of a passage might be to make an argument, many passages – even those consisting entirely of statements – fail to contain arguments. As a result, before you can apply techniques designed to evaluate arguments to a passage, you have to determine whether or not the passage even contains an argument.

The minimal condition that needs to be satisfied for a passage to contain an argument is for it to include at least two statements – a premise and a conclusion. But, in addition, these statements have to be structured in the right way. In particular, the passage has to include an inferential claim to the effect that the premises provide reasons or evidence for the conclusion.

There are three sorts of considerations that come into play when determining whether a passage contains an argument. First, the presence of indicator terminology – premise or conclusion indicator words and expressions – suggests a passage makes an inferential claim and, hence, contains an argument.

Premise indicators include such expressions as “because,” “since,” and “given that”; and they indicate that a statement that immediately follows one of them is a premise in an argument. Conclusion indicators include such expressions as “hence,” “therefore,” and “it follows that”; and they indicate that a statement that immediately follows them is a conclusion in an argument. Consider the passage,

It is snowing. It follows that it is cold.

The occurrence of the expression “it follows that” signals that the passage contains the claim that the statement "It is snowing" is being offered as evidence for the statement that it is cold.

Second, if a passage lacks any indicator terminology but there is an inferential relationship between the statements in it, this counts as evidence that the passage makes an inferential claim and, so, contains an argument. Consider the following:

Fido is a dog. Fido is one of Mary’s pets and all of Mary’s pets are dogs.

Because, as a matter of fact, the statement that Fido is a dog follows from the statements that Fido is one of Mary’s pets and that all of Mary’s pets are dogs, we have evidence that the passage contains an argument.

And third, if you are still unsure whether a passage contains an argument, consider whether or not the statement you think might be the conclusion is controversial. If this statement is controversial – and, hence, would need reasons or evidence to get people to accept it – then this counts in favor of taking the passage to contain an argument. But if this statement is a matter of common knowledge – and, hence, is in need of no support – then this counts against taking the passage to contain an argument.

B. Non-Arguments

There are a number of different kinds of passages that fail to contain arguments. Being able to identify different kinds of non-arguments will help you identify those passages that do contain arguments.

I. Unstructured Passages

Unstructured passages are non-arguments that are easily identified as such. Not only do they lack any kind of inferential claim, but they also lack the kind of structure in which one statement is singled out as having some kind of special status. These passages include the following:

1. Statements of belief: A **statement of belief** is a passage whose point is to convey the speaker’s opinions about something, such as

> I believe that all university students need to take Critical Thinking in their first year. I believe that all university students need to take courses that require them to write essays. I believe that all university students should be comfortable using quantitative methods.

2. Loosely associated statements: A passage made up of **loosely associated statements** consists of a collection of statements on the same general subject, such as

> Teachers need to work longer hours and spend more time leading extra-curricular activities. School maintenance budgets need to be cut without any reduction in services. Student test scores in core disciplines need to be substantially raised.

3. Reports: A **report** is a group of statements that convey information about some topic or event. The difference between a report and a collection of loosely associated statements concerns how tightly the statements in the passage are focused on a specific topic. The following passage contains a report:

> The subject was spotted loitering outside a convenience store. The officer approached the subject and asked him for identification. The subject was uncooperative and the officer took him into custody.

What is common to all of these types of passages is that there is no claim that any of their constituent statements supports or implies anything. For this reason, they count as non-arguments.

4. Conditional Statements: A **conditional statement** is a statement of the form "if … then…". The component statement immediately following the "if" is the *antecedent*; the component statement immediately following the "then" is the *consequent*. Conditional statements by themselves are not arguments – neither the antecedent nor the consequent is asserted – although they can be premises or conclusions of arguments. The following is an example of a conditional statement.

> If it is snowing then it is cold.

It is worth noting the contrast between this passage in the following.

> Since it is snowing, it is cold.

Because in the latter case the statement "it is snowing" is both asserted and offered as evidence for the statement "it is cold," this counts as an argument. But I could assert the former conditional statement confidently even if had no opinion as to whether or not it was snowing.

II. Structured Passages

The remaining types of non-arguments we will be discussing are often harder to distinguish from arguments. One reason for this is that, like arguments, they are structured: one of the statements has a special status and other statements have a role to play with respect to that special statement. But unlike arguments that role does not involve providing reasons or evidence for it.

5. Expository Passages: An **expository passage** is a collection of statements that begins with a topic sentence followed by one or more sentences that develop or elaborate on it. The role of the remaining statements is not to provide support for the topic sentence, but rather to expand or elaborate on it. And insofar as the objective is to expand/ elaborate rather than prove the topic sentence, there is no argument. The following is an example of an expository passage.

> Professor Jones has a very messy office. Her desk is entirely covered with papers, there is garbage on the floor, and there are coffee stains on the walls.

6. Illustrative Passages: An **illustrative passage** is a collection of statements providing one or more examples designed to show what something means. One of the statements in the passage expresses a generalization of some kind and the role of the remaining statements is to provide an instance of this generalization in order to illuminate it. If, however, the point of providing an instance of the generalization is to provide evidence for it rather than merely illuminating it, then the passage contains an argument by illustration rather than a nonargument. The following passage is an example of an illustration.

> University professors have advanced postgraduate degrees. For example, Professor Jones has a PhD in astrophysics.

7. Explanatory Passages: An **explanatory passage** consists of a group of statements that purports to shed light on some event or phenomenon. Like arguments, explanations are structured with one statement having a special status. In particular, the **explanandum** (the dumb one) is the statement that describes the event or phenomenon. The **explanans**, in contrast, is the statement or group of statements that purport to do the explaining. Moreover, explanations often contain the same kind of indicator terminology as arguments. The difference is in an argument, the truth of the conclusion is a matter of controversy and, as such, needs to be proven; but in an explanation the truth of the explanandum is taken for granted, but needs to be illuminated. The question in an explanation is not whether the explanandum is true, but rather why it is true. The following passage is an example of an explanation.

Acoustic guitars tend to develop cracks in Saskatoon. This is because the wood used to make guitars tends to crack in dry climates, and Saskatoon is fairly dry.

Exercises Section 1.4

Determine whether each of the following passages contains an argument. If it does, indicate the conclusion. If not, indicate what type of non-argument it is.

1. I believe that when it rains it pours. I believe that everything is beginning to unravel around us. I believe that the end is near.

2. Since either stinky cheese or steamed brussels sprouts have to be on the menu and the Anderson's are all allergic to the former, it follows that we're going to have a bland but healthy side dish.

3. Cattle produce huge quantities of methane gas. Methane has a very similar chemical composition to methanol. And drinking methanol can make you go blind.

4. I am suffering from an attack of gout in my big toe today. This is because I drank a case of beer last night and beer is high in purines.

5. Eating extremely spicy food can lead to indigestion. The theory that spices were originally used to make spoiled meat palatable has been thoroughly debunked. The theory that lightning never strikes the same place twice has also been debunked.

6. You should always treat others as you would have them treat you. For example, if you want other people to treat you with respect, you should treat them with respect as well.

7. If it rains it pours. And if it pours then water will seep into my basement. It follows that if it rains, water will seep into my basement.

8. If I consume food and drink containing high levels of purines and I neglect to take my gout medication, then I will be subject to attacks of gout in my big toe.

9. The key to a good curry is the careful preparation of the spice mixture. You need to make sure that you use fresh spices; you need to roast any whole seeds you use before you grind them up; and you need to make you measure out the spices accurately.

10. Jada called the meeting to order. Victor objected to the agenda on the grounds that his concerns from the previous meeting had not been adequately addressed. Trudy moved that the agenda be approved as is. No one agreed to second this motion. The meeting ended without a motion of adjournment.

11. Politicians are primarily motivated by self-interest. For example, Donald Trump used the US Presidency to promote his business interests. And Trudeau used his position as Prime Minister to secure luxury vacations.

12. The sky usually appears to be blue. This is because blue light from the sun is scattered by the molecules in the atmosphere to a much greater extent than light with longer wavelengths.

13. If Fred will go to the party only if Mary doesn't and Mary isn't going to go, then Fred will go to the party.

14. Since Fred will go to the party only if Mary doesn't and Mary isn't going to go, it follows that Fred will go to the party.

15. Getting ready for bed can be a fairly elaborate process. You need to change into pajamas, do your calisthenics, brush your teeth, apply your moisturizer, fill up your water glass, and make sure the doors are all locked, the stove is turned off, and none of the taps are dripping. Only then can you remove your housecoat and slippers, organize your sheets and pillows, place your book on your side table, and crawl under the covers.

Chapter 2: Argumentative Structure

1. Restructuring Arguments

In Chapter 1, we learned how to distinguish arguments from nonarguments and to distinguish between inductive and deductive arguments. But before we can begin developing methods for evaluating arguments, we need to learn how to identify the structure of arguments under consideration. In this chapter, we will be developing two techniques for doing just that. The first technique – restructuring – is best used for simpler arguments, whereas the second technique – diagramming – should be used to reveal the structure of more complex arguments.

Restructuring arguments involves identifying the premises and conclusion of an argument, labelling them P_1, P_2, P_3, ..., C to indicate their status as premises or conclusions, and listing them with the premises on the top and the conclusion on the bottom. Consider the following simple argument:

> All dogs are mammals, and all mammals are animals. Hence, all dogs are animals.

This argument would be restructured as follows:

> P_1: All dogs are mammals
> P_2: All mammals are animals
> C: All dogs are animals

The first thing you need to do in order to restructure an argument is to identify the premises and the conclusion. One key aid to doing so is the presence of **indicator terminology**. Indicator terminology consists of words and phrases that serve to indicate or identify the basic elements of an argument. **Conclusion indicators** provide clues for identifying the conclusion of an argument. Examples include the following:

therefore	so	as a result	it follows that
thus	hence	consequently	entails that

Premise indicators provide clues for identifying the premises of an argument. Examples include the following:

since for the reason that given that because

The rule of thumb here is that a statement that immediately follows a conclusion indicator is the conclusion of the argument and a statement that immediately follows a premise indicator is a premise in the argument. It should be emphasized, however, that because this is a rule of thumb and not an exceptionless formula, it cannot be applied mechanically. When you encounter indicator terminology, this counts as good, but not decisive, evidence for taking a statement to be a premise or a conclusion. As a result, in any given case in which you encounter such terminology, you need to stop and think about whether or not the statements indicated really do play one or the other of these roles in the broader passage at issue. Note: if a passage lacks any indicator terminology, you need to ask yourself what the main point of the passage is, that is, what the arguer is trying to convince you of; this will help you determine which statement counts as the conclusion of the passage, which will make it correspondingly easier to identify the premises.

Once you have identified the premises and the conclusion of an argument under consideration, there are a number of guidelines you need to keep in mind as you begin to restructure it. First, you need to make sure that you write the premises and conclusion in the form of separate declarative sentences. Suppose a passage included the following language:

> Wine coolers are better than whiskey? Not!

Rather than incorporating this into your reconstruction as

> P_3: Wine coolers are better than whiskey? Not!

you should instead rewrite it as

> P_3: Wine coolers are not better than whiskey,

or something along these lines. Note: when restructuring arguments, it is always preferable to stay as close to the original language of the passage as possible.

Second, in the restructured version of the argument, omit any indicator terminology that occurs in the passage. Consider the follow argument:

Since it is snowing, it follows that it is cold.

Rather than restructuring this as

P_1: Since it is snowing

C: It follows that it is cold

you should instead restructure it as

P_1: It is snowing

C: It is cold.

After all, the labels P_1 and C take over the job of indicating whether statements are premises or conclusions played by the indicator terminology in the original passage.

Third, if a passage contains a compound statement – one which contains two simpler statements as components – and both of its components are claimed to be true, then these components should be treated as separate statements in the reconstructed argument. For example, if a passage contains the following compound statement as a premise,

Professor Alward is a cad and cads should be prohibited from university classrooms,

then it should be reconstructed as

P_1: Professor Alward is a cad

P_2: Cads should be prohibited from university classrooms

rather than as

P_1: Professor Alward is a cad and cads should be prohibited from university classrooms.

Doing this makes more of the structure of the argument apparent which will help facilitate its evaluation. Note: even though they contain two statements as components, “either … or …” and “if … then …” statements should not be broken up in argument reconstructions.

Fourth, some passages include extraneous statements which are neither premises nor the conclusion of the argument it contains. An example might be the following:

> The watercooler dispute over today’s temperature has gone on long enough. Since it is snowing, it follows that it is cold.

Since the first sentence of this passage merely introduces the topic without serving as a premise or conclusion in the argument which follows, it should not appear in the reconstructed argument. As a result, the correct reconstruction of this argument is

> P_1: It is snowing
>
> C: It is cold.

and *not*

> P_1: The watercooler dispute over today’s temperature has gone on long enough
>
> P_2: It is snowing
>
> C: It is cold.

The guidelines for reconstructing arguments can be summarized as follows:

> 1. List the premises first (labelled P1, P2, P3, ...) and the conclusion (labelled C) last.
> 2. Write the premises and the conclusion in the form of separate declarative sentences.
> 3. Eliminate all premise and conclusion indicators.
> 4. Compound arrangements of statements in which the various components are all claimed to be true will be considered separate statements.
> 5. Omit statements that are neither premises nor conclusions.

Finally, let us run through one last example. Suppose we want to reconstruct the following argument:

> You might think that online courses are better than face-to-face courses, but you'd be mistaken. Student evaluations are stronger in face-to-face courses and grade averages are higher too. And since higher grades reflect better learning outcomes, it follows that face-to-face courses are superior.

The first thing we need to do is to identify any indicator terminology. One way to keep track of it is to put it inside parentheses as below:

> You might think that online courses are better than face-to-face courses, but you'd be mistaken. Student evaluations are stronger in face-to-face courses and grade averages are higher too. (And since) higher grades reflect better learning outcomes, (it follows that) face-to-face courses are superior.

Second, we need to identify any extraneous statements. Since they will not appear in our reconstruction, we can simply cross them out:

> ~~You might think that online courses are better than face-to-face courses, but you'd be mistaken~~. Student evaluations are stronger in face-to-face courses and grade averages are higher too. (And since) higher grades reflect better learning outcomes, (it follows that) face-to-face courses are superior.

Third, we need to identify any compound statements that need to be broken up. For convenience, we can put the components of compound statements that will appear in our reconstruction inside square brackets as follows:

> ~~You might think that online courses are better than face-to-face courses, but you'd be mistaken~~. [Student evaluations are stronger in face-to-face courses] and [grade averages are higher too]. (And since) higher grades reflect better learning outcomes, (it follows that) face-to-face courses are superior.

Once we have completed our preliminary analysis of the passage, we can easily complete our reconstruction of the passage:

> P_1: Student evaluations are stronger in face-to-face courses
> P_2: Grade averages are higher too
> P_3: Higher grades reflect better learning outcomes
> C: Face-to-face courses are superior

Exercises Section 2.1

Restructure each of the following arguments.

1. Public education is one of the most expensive items in the provincial budget. And the people who benefit most directly from it pay little or nothing by way of taxes. As a result, public schools should be turned into workhouses for poor children.

2. Since our current crop of politicians is primarily motivated by greed, we need to vote them out in the next election. Moreover, it’s a good idea to change the party in power every election anyhow.

3. The Raptors need to trade for a new starting center. This is because the current centers on the roster aren’t strong enough to defend against their bigger counterparts on other teams and their likely draft position is too low to acquire an adequate big man through the draft.

4. Johnny Depp has been proven to be guilty of spousal abuse, so Hollywood studios should refrain from hiring him to star in their movies. In addition, his contract demands are too high for his movies to be profitable.

5. People who refuse to get vaccinated pose a risk to the health of other people. But the fewer anti-vaxxers there are, the lower the risk they create. So people should be allowed to refuse to take vaccines for diseases with high mortality rates.

6. Logic classes are boring and logic tests are too hard. As a result, you should drop logic. After all, if the tests in a course are too hard, you’ll end up with a bad grade.

7. Face-to-face courses are the worst. You either have to get up early or you’ll be late for class. But I hate getting up early and professors are always snarky when you’re late.

8. Any animal which demands too much of your attention makes a bad pet. So, since cats are more independent than dogs and their excrement is not spread all over your yard, cats make much better pets than dogs.

9. Regular showering is the best way to keep clean. Baths involve wallowing in your own filth and they use too much hot water. And reducing our energy use helps in the fight against global warming.

10. Bad guitar playing is really annoying. The only way to avoid it is to require guitar players to get licenses before playing in public or to ban public guitar playing altogether, but licensing guitar playing would involve government overreach. As a result, we need a complete ban on public guitar playing.

11. A lot of people think that laws prohibiting public urination are justified, but they're just wrong about that. People often can't make it home before they have to go and a lot of businesses won't let you use their washrooms unless you buy something first. And where are homeless people supposed to go anyhow? Moreover, if people cannot see you go, they have no reason to complain. Hence, as long as they're discreet about it, people should be allowed to urinate wherever they like.

12. There's a lot of talk of defunding the police in the media these days. But if we defund the police, we're going to have to fire a lot of police officers and replace them with social workers. The result would be a smaller force of officers dealing with the same number of crimes, and overworked and demoralized employees are always less effective at their jobs. Are the social workers supposed to pick up the slack? The conclusion is clear: the police should not be defunded.

13. If we re-open the border with the US, our COVID-19 case numbers will spike. This is because of the large numbers of COVID-denying anti-maskers in the US. Some people think these people wouldn't want to come to Canada anyhow, but that's just wrong. Why wouldn't they? If they did come, they would ignore our mask use and social distancing policies and there's no better way than that to cause our numbers to spike.

14. Since social distancing cannot be maintained in U of S classrooms, we should hold our face-to-face classes in the Bowl. There's plenty of room to spread out in the Bowl and there are no worries about ventilation. Some people think that once we hit November, it would be too cold to hold classes outside. Don't they realize U of S students are used to the cold? The conclusion is obvious.

15. Lots of people think hoarding should be discouraged but, in my view, it's time to rethink our societal bias against hoarding. We can either keep our stuff or send it to the landfill, but burying our garbage is unsustainable. As a result, hoarding should be encouraged. Some people think we should just stop buying so much junk. But retail shopping is the backbone of the Canadian economy. Do they want to put all those people out of work?

2. Argument Diagrams

Restructuring is an effective technique for revealing the structure of simpler arguments, but it falls short when applied to more complex arguments. Some arguments contain sub-arguments for one or more of the premises used to support the main conclusion. Other arguments have multiple conclusions. And distinct premises in an argument sometimes work together and sometimes work independently to provide support for the argument's conclusion. The technique of restructuring, however, lacks the resources to represent any of these features of arguments. And this is where the technique of diagramming comes into play: it is specifically designed to represent these features of arguments.

The first step in the diagramming procedure is to number the statements that count as premises and conclusions of the argument contained in a passage under consideration, normally in the order in which they occur in the passage. Consider, again, the following argument:

> Since it is snowing, it follows that it is cold.

We would number the statements in this argument as follows:

> Since (1) it is snowing, it follows that (2) it is cold.

Note: since indicator terminology merely identifies premises and conclusions and is not part of the content of such statements, we place the numbers to the right of such expressions. As a result, the following numbering of our passage would be *incorrect*:

> (1) Since it is snowing, (2) it follows that it is cold.

It is also important to note that statements which do not serve as premises or conclusions in the argument contained in a passage do not receive numbers. As a result, the correct numbering of a passage such as,

> The watercooler dispute over today's temperature has gone on long enough. Since it is snowing, it follows that it is cold.

would be,

> The watercooler dispute over today's temperature has gone on long enough. Since (1) it is snowing, it follows that (2) it is cold.

and *not*,

> (1) The watercooler dispute over today's temperature has gone on long enough. Since (2) it is snowing, it follows that (3) it is cold.

Once we have numbered the various statements in an argument under consideration, we are ready to create a diagram for that argument. The most fundamental element of an argument diagram is the arrow. When an arrow points from one number to another in an argument diagram that means that the statement designated by the number the arrow is pointing from is claimed to provide reasons or evidence for the statement the arrow is pointing at. Consider, again, the following argument:

> Since (1) it is snowing, it follows that (2) it is cold.

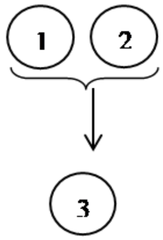
The indicator terminology in this argument – "since" and "it follows that" – tells us that statement (1) is claimed to provide reasons or evidence for statement (2). As a result, the correct diagram for this argument would be the following:

As should be evident, indicator terminology will play a central role in the procedure of diagramming arguments.

When multiple premises are claimed to support a single conclusion, they can either be conjoint or independent. To say that two or more premises are conjoint is to say that if taken separately, they provide little or no support for the conclusion but taken together they do provide support. We represent this in an argument diagram by means of a single arrow from a brace encompassing the various premises to the conclusion. Consider the following argument:

> (1) If it is snowing then it is cold, but (2) it is not cold. It follows that (3) it is not snowing.

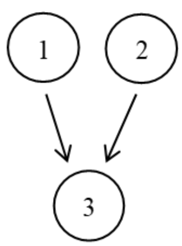
Since neither (1) nor (2) considered independently offer any support for (3), but taken together they do offer some such support, it follows that they are conjoint premises for (3). As such, the correct diagram for this argument is the following:



To say that two or more premises are independent, in contrast, is to say that each would continue to provide the same degree of support for the conclusion if the others were omitted. Indicator terminology, such as “moreover” or “in addition,” may also suggest that what follows is an independent reason for a conclusion. We represent this in an argument diagram by means of a separate arrow from each of the premises to the conclusion. Consider the following argument:

> (1) Logic courses are hard and (2) logic professors lack interpersonal skills. As a result, (3) you should drop logic.

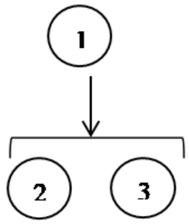
Since each of (1) and (2) would continue to provide support for (3) if the other were omitted, it follows that they are independent premises for this conclusion. As such, the correct diagram for this argument is the following:



Now not only can multiple premises support a single conclusion, in some arguments a single premise can support multiple conclusions. We represent this in an argument diagram by means of a single arrow from the premise in question to a brace encompassing the various conclusions supported by it. Consider the following argument:

> (1) It is sleeting out; therefore (2) it is cold and (3) it is wet.

Since (1) is offered as support for both (2) and (3), it follows that we have a case of a single premise supporting multiple conclusions. As such, the correct diagram for this argument is the following:



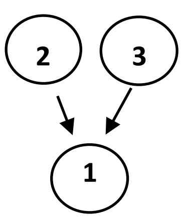
Finally, let's review by running through the diagramming process from start to finish with the following more complex example.

> "Logic is hard because the symbols are indecipherable, and the rules don't make any sense. So, no one should take logic courses. Moreover, logic professors have poor personal hygiene and students shouldn't be forced to take courses from malodorous instructors."

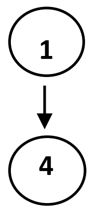
The first step is to number the statements that count as premises or conclusions in the argument. For convenience, I will highlight indicator terminology at the same time.

> "(1) Logic is hard because (2) the symbols are indecipherable, and (3) the rules don't make any sense. So (4) no one should take logic courses. Moreover, (5) logic professors have poor personal hygiene and (6) students shouldn't be forced to take courses from malodorous instructors."

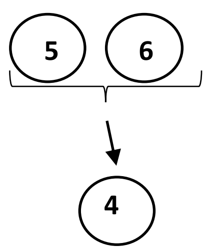
The term "because" in the first sentence tells us that (2) and (3) are offered as reasons for (1). Moreover, since each of them would provide support for (1) even if the other were omitted, they count as independent premises for this conclusion. Hence, we can diagram this part of the argument as follows:



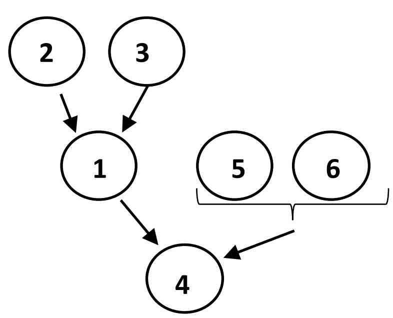
The term “so” in the second sentence tells us that (4) follows from (1), the conclusion of the sub-argument that precedes it. As a result, we know that the argument will include the following:



The term “moreover” at the start of the third sentence sentences indicates what follows is an independent set of reasons for (4), the main conclusion of the argument. But since neither (5) nor (6) would provide support for (4) if the other were omitted, they count as conjoint premises for this conclusion. Hence, we can diagram this part of the argument as follows:



Finally, if we combine the diagrams for the various parts of the argument, we get



which is the complete diagram for the whole argument.

Exercises Section 2.2

Diagram each of the following arguments.

1. (1) It’s warm out because (2) it’s sunny and (3) whenever it’s sunny it’s warm.

2. (1) Walking around without your shoes on is undignified and (2) U of S classroom floors are slippery therefore (3) teaching in sock feet is a bad idea.

3. (1) The end is near because (2) Alward predicted a calamity would befall humanity sometime this year. It follows that (3) we should get our affairs in order.

4. (1) Your sports jacket is covered in vomit therefore (2) you cannot wear it. And since (3) you can attend the Dean’s reception only if you wear your sports jacket, it follows that (4) you cannot go to the reception.

5. (1) You can either stay in bed watching Netflix or go to the library to study because (2) class has been cancelled today. But since (3) the library is closed, (4) you cannot go there to study. Therefore (5) your only option is to spend the day in bed watching Netflix.

6. (1) It seems like you are looking through a window when you watch *Domicile’s Quiver* therefore (2) it is the most realistic film ever made and (3) the actors behave just like real people. Since (2) it is the most realistic film ever made, (4) the cinematography in *Domicile’s Quiver* is beyond compare; and (5) the acting was *Domicile’s Quiver* is extraordinary because (3) the actors behave just like real people. For these reasons (6) *Domicile’s Quiver* deserves to win an Oscar.

7. (1) The sky is falling because (2) if it’s not, I’m having a nervous breakdown and (3) my mental health is just fine. (4) If the sky is falling, people are going to be looking for ways to protect their heads and (5) there’s no better head protection than a good helmet, therefore (6) if the sky is falling then the protective helmet industry is going to take off. As a result (7) it’s time to invest in Edie’s Helmets, Inc.

8. (1) Mabel really hates Arthur. Hence since (2) she makes a point of avoiding people she hates, (3) if Arthur signs up for a class, Mabel will not sign up for it. But since (4) Mabel has already signed up for yoga, (5) Arthur must have decided not to. And since (6) yoga seems to be the only exercise Arthur ever gets, (7) his health is going to deteriorate even more this year.

9. (1) Not washing your hands regularly can spread germs. In addition (2) it’s just gross. And for these reasons (3) you need to wash your hands several times a day. Hence, (4) you’re going to need to purchase a lot more hand soap. Moreover, because (5) the zombie apocalypse is coming and (6) in a post-apocalyptic world, luxury items will be increasingly hard to find, (7) there’s soon going to be a soap shortage. And that is another reason (4) you’re going to need to purchase a lot more hand soap.

10. (1) You need to go to bed earlier because (2) you can't continue being late for work all the time. Moreover, (3) you'll be more effective at your job if you're well-rested and (4) the best way to get more rest is to go to bed earlier. Consequently, since (5) binge-watching procedural police dramas is what keeps you up so late, (6) you're going to have to change your streaming habits. Therefore (7), from now on you should only watch light comedies.

11. (1) You're subject to frequent attacks of gout because (2) you're constantly bedridden with swollen toes; (3) your kids thought you were a ghost hence (4) you have an unhealthy pallor; and since (5) the TV has been tuned to the infomercial channel for the past week, (6) you barely have the strength to change the channel on the TV remote. For these reasons, (7) you need to eat better and (8) you need to start exercising.

12. (1) If Kyle Lowrie joins our book club, then Mary will start bringing fancy beer and (2) if Mary brings fancy beer, then Fred will start bringing fancy hors d'oeuvres. Hence, since (3) if Fred starts bringing fancy hors d'oeuvres then Lebron James will want to join, it follows that (4) if Kyle Lowrie joins our book club, then Lebron James will want to join as well. But because (5) Lebron James won't join any club Meryl Streep is a member of and (6) and Meryl Streep is a member is a member of our book club, (7) Lebron James won't join. Therefore, (8) Kyle Lowrie won't join either.

13. (1) Janice is going to be upset because (2) she lost her job and (3) her brother got arrested. Hence since (4) Rashid likes to console his friends and (5) Janice is a friend of his, (6) Rashid going to be busy consoling Janice for the next week or so. And since (7) the deadline for the hurricane machine proposal is in four days and (8) no one other than Rashid is competent to complete the proposal, it follows that (9) your hurricane machine project will have to be abandoned.

14. (1) If I get out of bed I will have to take a shower, (2) if I take a shower I will have to get dressed, (3) if get dressed I will have to have breakfast, and (4) if I have breakfast I will have to go to work. But (5) I don't want to go to work. So (6) I'm not going to get out of bed. But (7) if I don't get out of bed, I'll lose all my friends, and (8) if I lose all my friends, I'll have to acquire new interests. Hence (9) I'll have to spend the day in bed researching bed-compatible hobbies.

15. (1) S/he who laughs last laughs longest. This is because (2) s/he who laughs second last will be interrupted by the last laugher, (3) the last laugher will not be interrupted by any other laughers, and (4) uninterrupted laughing can go on indefinitely. Therefore, since (5) you want to have the best laugh and (6) the best laugh is the longest laugh, it follows that (7) you need to laugh last. And since (8) the best way to laugh last is to get others to laugh, (9) you need to tell lots of jokes.

Chapter 3: Informal Fallacies

A good argument is one in which the premises provide adequate support for the conclusion. As noted in Chapter 1, what counts as adequate support depends on the type of argument at issue: in a deductive argument the truth of the premises must guarantee the truth of the conclusion; in an inductive argument, the premises need only make the truth of the conclusion sufficiently probable. Sometimes, however, bad arguments – in which the premises do not provide adequate support for the conclusion – are nevertheless persuasive. A **fallacy** is a type of bad argument that has proven to be regularly persuasive, that somehow creates an illusion that serves to make it *seem* good. A large number of fallacies have been catalogued over the years: in this chapter we are going to consider a sample of the more common fallacies you are likely to encounter in your personal and professional endeavors. Although we will not be sorting each of the fallacies we consider into a strict system of categories, it will prove useful to identify certain clusters of fallacies.

1. **Ad Hominem (Against the Person)**: One of the most well-known fallacies is the Ad Hominem fallacy, sometimes called the "argument against the person." This fallacy occurs when one person advances an argument and another person responds by directing his or her attention, not to the argument itself, but to the person who made it. Suppose, for example, that the following interchange occurs:

> Smith: Radio signals, with extra-terrestrial origins, have been detected which strongly indicate the presence of intelligent alien life.
>
> Jones: Smith is a foul-smelling buffoon whose views can be safely ignored.

Rather than addressing the quality of Smith's argument, Jones instead focuses on Smith's personal attributes. As a result, Jones has given no grounds for rejecting Smith's argument. More generally, except in special circumstances in which the personal attributes of the subject of the ad hominem attack are relevant to the claim that is at issue – whether s/he is a reliable witness, e.g. – a response of this kind is fallacious.

There are a number of distinct versions of the ad hominem fallacy, three of which will be discussed here. First, the **abusive** ad hominem occurs when a respondent uses abusive language against their argumentative opponent. An example would be the following:

> Trudeau says we should have gun control. But he's a scumbag. Hence, Gun control is not permissible.

Second, the **circumstantial** ad hominem occurs when a respondent accuses their argumentative opponent of having a personal stake in the outcome of the dispute which entails that the latter’s argument should not be taken seriously. An example would be the following:

> Singh supports mandatory union membership. But Singh has an interest in courting the union vote. Hence, mandatory union membership is wrong.

And third, the **tu quoque** (or hypocritical) ad hominem occurs when a respondent attempts to make their argumentative opponent appear to be hypocritical or arguing in bad faith, that is, engaging in behavior which conflicts with his or her stated conclusion. An example would be the following:

> Dr. Smith says smoking is dangerous, but Dr. Smith smokes. Hence, smoking is not dangerous.

Cluster I: Appeals to Emotion (ad baculum, ad misericordiam, ad populum)

2. **Ad Baculum (Appeal to Force)**: The Ad Baculum fallacy or the “appeal to force” occurs when an arguer poses a conclusion to a disputant and tells that person, either implicitly or explicitly, that some harm will come to him or her if he or she does not accept the conclusion. In effect, this fallacy involves threatening someone’s psychological or physical well-being in order to get them to adopt one’s point of view. It is a fallacy because it attempts to get someone to accept a position out of fear rather than by providing reasons or evidence for that position. Consider the following example:

> My restaurant deserves a AAA health rating. Vito Corleone [the “godfather”] thinks my restaurant is the cleanest he’s ever been in.

This argument consists of an implicit threat of violence from a mafia don if the person to whom it is directed does not accept the conclusion. And the recipient of the threat might actually do so out of fear. But even if s/he does, no reasons or evidence has been offered for the claim that the restaurant in question has met the relevant health standards.

3. **Ad Misericordiam (Appeal to Pity)**: The Ad Misericordiam fallacy, or the "appeal to pity," occurs when an arguer attempts to support a conclusion by merely evoking pity from the reader or listener. In effect, the goal is to use the suffering the arguer would undergo if their thesis were not accepted to persuade the listener to accept that thesis. It counts as a fallacy because the (supposed) fact that one would suffer if one's claim was not accepted provides no reason for thinking that claim is correct. Consider the following example:

> Freddie Jones and his children would be distraught if Josie McPhee were imprisoned. Moreover, they would lose their home and become impoverished. She really doesn't deserve to go to jail.

The argument is designed to evoke pity in the listener in order to get this listener to accept that Josie McPhee does not deserve jailtime. But the putative fact that McPhee's family would suffer if she were imprisoned provides no reason to suppose she does not deserve to be imprisoned (although it might count as evidence that she should not in fact be imprisoned, whether or not she deserves it).

4. **Ad Populum (Bandwagon)**: The Ad Populum or "bandwagon" fallacy occurs when an arguer uses people's desires to be loved, accepted, etc. to get listeners to accept a conclusion. In its most basic form, it involves an appeal to the fact that members of a desirable group accept a claim to get the listener to accept it as well. It counts as a fallacy because, except in special cases – when the issue is what people believe, e.g. – the fact members of a target class believe a claim provides no reason to think it is correct. Consider the following example:

> The Kardashians and their entourage believe that the world is flat. You should too.

This argument is designed to manipulate the listener's admiration for the Kardashians in order to get them to adopt unorthodox beliefs about the shape of the planet. But, again, no evidence is offered to support this claim.

Cluster II: Parts and Members (accident, hasty generalization, composition, division)

5. **Accident (Destroying the Exception)**: The Accident fallacy, or "destroying the exception," occurs when a general rule is applied to a specific case it was not intended to cover. Some generalizations are intended to apply only to a subclass of a broader population of entities of some kind. A fallacy occurs when it is

nevertheless applied to members of the population that fall outside of this subclass. Consider the following example:

> Regular exercise is an essential part of a healthy lifestyle. Hence, a diagnosis of pneumonia isn't going to get you out of gym class.

The generalization linking exercise to a healthy lifestyle only applies when the person in question satisfies a minimal condition of health or fitness. But whatever this minimal condition is, someone suffering from pneumonia does not satisfy it. The fallacy consists in applying the generalization to this person regardless.

6. **Hasty Generalization**: The Hasty Generalization fallacy is, in effect, the converse of the Accident fallacy. It occurs when a too small or unrepresentative sample of a population is used to justify a generalization about all or most members of that population. It is a fallacy because it yields a weak inductive argument – one whose premises leave the conclusion unlikely. Consider the following argument:

> Professor Alward's family members think he would make the best Prime Minister. It follows that Trudeau should worry about losing to him in the next election.

Not only is Professor Alward's family too small to draw any conclusions about the population of Canada, but their views about the relative merits of Alward and Trudeau are unlikely to be representative of the broader population. The fallacy consists in going on to make the generalization despite the inadequacy of the sample.

7. **Composition**: The fallacy of Composition occurs when the conclusion of an argument depends on the erroneous transference of an attribute from the parts of something onto the whole. In some case, it is legitimate to argue that what is true of the parts of a thing is also true of the whole thing itself. Consider the following example:

> Each of the bricks which entirely constitute the wall is made up of 50% silica. It follows that the wall itself is made up of 50% silica.

Because the material make-up of each of a collection of bricks and of a wall made entirely out of those bricks is the same, the transference of this feature of the bricks onto the wall is entirely appropriate. But in other cases, this is illegitimate. Consider, for example,

> Each of the bricks that wholly constitute the wall can withstand an earthquake of up to 7.5 on the Richter scale. Therefore, the wall itself can withstand an earthquake of up to 7.5 on the Richter scale.

Because the earthquake-resistance of an individual brick cannot be assumed to be the same as a wall made out of those bricks, the transference of this feature of the bricks onto the wall is inappropriate. More generally, if you notice that an argument involves a transfer of attributes from the parts of a thing onto the whole, you should consider it to be a candidate for having committed the fallacy of composition. But then you have to bring your knowledge of the details of the case to bear in order to determine whether or not this fallacy has in fact been committed.

8. **Division**: The fallacy of Division is, in effect, the converse of the fallacy of composition. It occurs when the conclusion of an argument depends on the erroneous transference of an attribute from a whole onto its parts. As with composition, it is sometimes legitimate to argue that what is true of a whole thing is also true of the parts that make it up. Consider the following example:

> The wall is made up of 50% silica. It follows that each of the bricks which entirely constitute the wall is also made up of 50% silica.

As above, since the material make-up of a wall and of each of the collection of bricks that wholly constitute it is the same, the transference of this feature of the wall onto the bricks is entirely appropriate. But in other cases, this is illegitimate. Consider, for example,

> The wall cannot withstand an earthquake of 7.5 on the Richter scale. It follows that none of the bricks that wholly constitute the wall can withstand an earthquake of 7.5 on the Richter scale.

As above, since the earthquake-resistance of a wall and of the individual bricks that wholly constitute it cannot be assumed to be the same, the transference of this feature of the wall onto the bricks is inappropriate. But more generally, if you notice that an argument involves a transfer of attributes from a whole thing onto its parts, you should consider it to be a candidate for having committed the fallacy of division and bring your knowledge of the case to bear.

Cluster III: Changing the Subject (straw person, irrelevant conclusion, red herring)

9. **Straw Person**: The Straw Person fallacy occurs when an arguer distorts an opponent's argument for the purpose of more easily attacking it. Like the Ad Hominem fallacy, the Straw Person arises as a response to another person's argument. But instead of attacking his or her argumentative opponent's person or character, in this case, the respondent weakens and misrepresents the opponent's position. Consider the following argument:

> Professor Alward says that Nova Scotians should be allowed to teach in Saskatchewan classrooms. Clearly, he thinks that Saskatchewan students need to learn about cod fishing from people with funny accents. But that is the last thing students in the prairies need to learn.

In this example, the respondent does not address his or her argumentative opponent's claim that Nova Scotians should be allowed to teach in Saskatchewan classrooms. Rather the respondent recasts it as the claim that Saskatchewan students need to learn about cod fishing from people with funny accents and rejects this distorted version of his or her opponent's views. This counts as a fallacy because proving the falsity of the distorted claim does not in anyway show that the actual claim – Nova Scotians should be allowed to teach in Saskatchewan classrooms – is incorrect.

10. **Irrelevant Conclusion** (**Missing the Point)**: The fallacy of Irrelevant Conclusion, or "missing the point," occurs when the premises of an argument support one conclusion but a different, often vaguely related, conclusion is drawn. This counts as a fallacy because the conclusion that is actually drawn is not supported by the premises on offer, despite the erroneous appearance that they might do so. Consider the following example:

> Theft and robbery have been increasing at an alarming rate. Hence, we should reinstate the death penalty.

The premise of this argument – that theft and robbery have been increasing at an alarming rate – might well support the conclusion that the penalties for such crimes need to be made more severe. But theft and robbery are not potential death penalty offenses, at least not in Canada. As a result, attempting to use increasing theft and robbery rates to support reinstating the death penalty is fallacious.

11. **Red Herring**: The Red Herring fallacy occurs when the arguer diverts the attention of the listener by changing the subject to a different but subtly related

one. This counts as a fallacy because the considerations raised relate only to the new topic despite the erroneous appearance that they concern the original topic at issue. Consider the following example:

> The statistics showing seatbelts make you safer are flawed. After all, forcing people to use seatbelts is an infringement of liberty. It's the first step to authoritarian government.

It may or may not be true that mandatory seat seatbelt legislation is an unjustified infringement of liberty. But that is a separate question from whether or not seatbelts make your safer. So even if it is true that seatbelt legislation infringes liberty, that in no way shows that seatbelt use fails to improve safety. Note: the way to distinguish between an argument that makes the irrelevant conclusion fallacy and one that makes the red herring fallacy is by appeal to where the conclusion (or the statement of the issue) appears in the passage. In the irrelevant conclusion fallacy, the erroneous conclusion occurs at the end of the passage. In the red herring, in contrast, the conclusion occurs upfront, after which the topic is changed to a subtly related one.

Cluster IV: Weak Induction (ad ignorantiam, slippery slope)

12. **Ad Ignorantiam (Appeal to Ignorance)**: The Ad Ignorantiam fallacy, or the "appeal to ignorance," occurs when the premises of an argument establish that a thesis of some kind has not been proven and, on that basis, it is concluded that the contrary must be correct. Except in cases in which it is established that if the thesis were true, we should expect to find certain evidence that we in fact have not found, this style of reasoning is fallacious. After all, the mere absence of evidence for a thesis by itself does not prove anything at all. Consider the following example:

> No one has ever proved that ghosts don't exist. We can therefore conclude that they do exist.

Suppose that the premise is true and that the nonexistence of ghost has not been established. In the absence of any positive evidence for the existence of ghosts, the most this would establish is that one ought to adopt an attitude of neutrality on the question. The fallacy consists in taking to the lack of proof of the nonexistence of ghosts to count as positive evidence for their existence. This point can be made more compelling by noting that a completely analogous argument,

> No one has ever proved that ghosts do exist. We can therefore conclude that they don't exist,

can be used to draw the opposite conclusion.

13. **Slippery Slope**: The Slippery Slope fallacy occurs when the conclusion of an argument rests upon an alleged chain reaction when there is not sufficient reason to think the chain reaction will occur. Sometimes, if a proposition of some kind were true, there would be consequences of various kinds, consequences that may not have in fact occurred or are best avoided. So, for example, if the proposition that the legal drinking age should be lowered to 12 were accepted, and the law was changed accordingly, this could seriously impact the cognitive development of children. And this would count against accepting the proposition that the drinking age should be lowered. But an argument of this kind is only as good as the likelihood that the consequences of accepting the proposition would occur. And when the consequences are supposed to arise only as a result of an unlikely sequence of unlikely events, this makes the argument all the worse. Consider the following example:

> If I had stolen your identity, I would have emptied your bank account and purchased all of the vintage comic-books in the city. This would have caused outrage among comic-book enthusiasts, which would have led to comic book riots. The result would have been the destruction of all the comic-book stores in the city. But as you well know, there are lots of comic-book stores still open in the city. It follows that I did not steal your identity.

The argument claims if the proposition that I stole your identity were true, the consequence would have been the destruction of all the comic-book stores in the city; and the fact that this has not come to pass is thereby supposed to establish that this proposition is false. But given that the destruction of the city's comic-book stores is an unlikely consequence of an unlikely sequence of events, this argument is fallacious.

Cluster 5: Presuppositions (loaded question, false dilemma)

14. **Loaded Question (Complex Question)**: The Loaded Question fallacy occurs when a question is posed which contains a controversial presupposition. As a result, whatever answer the listener gives to the question implicitly affirms a proposition the listener might not accept. Consider the following example:

> Are you still plagiarizing your coursework?

This question presupposes that the listener has in the past been plagiarizing his or her coursework. After all, if the listener answers “yes,” that entails that s/he is continuing a past practice of plagiarism; and if s/he answers “no,” that entails that s/he has discontinued a past practice of plagiarism. Now if there is no controversy about the listener’s previous plagiarism – if s/he has confessed, e.g. – then there is nothing fallacious about the question. But if the charge of plagiarism is controversial – if the listener denies it, e.g. – then the question is fallacious.

15. **False Dilemma (False Dichotomy)**: The False Dilemma fallacy, or “false dichotomy,” occurs when an either/or premise is deployed which presents two unlikely alternatives as if they were the only ones available. The arguer then provides grounds to eliminate one of the options, leaving his or her preferred option as the only one still standing. This form of argument is sound as long as the either/ or premise is true, that is, as long as it includes all of the genuine options. But when there are other, more likely, options not included in this premise, the argument is fallacious. Consider the following example:

> Either I deserve the Noble Prize for Chemistry or the chemical laws have changed since I published my seminal work. But the laws of nature are static. It follows that I deserve the Noble Prize.

If the only (reasonably likely) options really were that the laws of nature had changed or that the arguer deserves a Nobel Prize, then the argument would be a good one. But since there are other more likely options not included in the either/or premise – most notably that the arguer’s work was not of sufficient significance to merit such an award – the argument is a fallacy.

Cluster VI: Ambiguities

16. **Equivocation (Semantic Ambiguity)**: The fallacy of Equivocation, or “semantic ambiguity,” occurs when the conclusion of an argument depends on the fact that a word or phrase is used in two different senses in the argument. Whether or not the premises of an argument are both true and provide adequate support for the conclusions depends in part on the meanings of the expressions included in them. And in order for an argument to establish its conclusion, each occurrence of an expression has to be understood to have the same meaning, both when adjudicating the truth of the premises and when assessing the degree of support they provide for the conclusion. An argument is fallacious when it

depends on one or more expressions being used in different senses. Consider the following example:

> Crows' feathers are extremely light. Hence, that black bird cawing in the tree cannot be a crow.

The word "light" is used in two different senses in this argument. In order for the premise to be true, "light" has to be understood to mean "not heavy," but in order for the conclusion to follow from the premises it has to be understood to mean "pale in colour." As a result, this argument is fallacious.

17. **Syntactic Ambiguity (Amphiboly):** The fallacy of Syntactic Ambiguity, or "amphiboly," occurs when the conclusion of an argument depends on the fact that a premise or conclusion is ambiguous between two or more grammatical structures. Typically, this involves arguments in which the truth of the ambiguous statement requires interpreting it as having one grammatical structure while the inference from the premises to the conclusion requires taking it to have another grammatical structure. Consider the following example:

> Officer Smith arrested a suspect with an open bottle of whiskey in his hand. It follows that Officer Smith ought to be reprimanded for drinking on the job.

The premise of this argument is ambiguous between a reading on which the suspect has an open bottle of whiskey in his hand and a reading on which Officer Smith has the bottle of whiskey in his hand. The conclusion follows from the premise only if the premise is interpreted to mean that Smith has the bottle; but the premise is true only if it is interpreted to mean that the suspect has the bottle. As a result, the argument is fallacious.

Exercises Section 3.1

Give the name of any fallacy that occurs in the following arguments. If the argument contains no fallacy, say so.

1. Ignatius McDingle claims that organic farming techniques are better for the environment. But he is an ill-mannered, foul-smelling reprobate. His views should simply be ignored.

2. The claim that wearing socks with sandals is unstylish should be rejected. Sock manufacturers and retailers are struggling in the current economy. Any

further decreases in sales during “Sandal Season” will drive many of them out of business.

3. Elitist snobs would have you believe that so called “craft beers” taste better. But regular Joes know that nothing tastes better than Duff Beer. Be a regular Joe: drink Duff.

4. If I had lied to you about my whereabouts last night, that would have thrown the moral balance of the universe out of whack. This would have caused the universe to attempt to restore this balance by generating an increase in good deeds. And that would have caused the woman in the Motörhead t-shirt to thank you when you returned her keys to her. But she didn’t thank you, did she? That proves I didn’t lie to you.

5. Ignatius McDingle claims that organic farming techniques are better for the environment. His view seems to be farmers should just let pests run wild over their crops. But that would result in crop losses on an unprecedented scale, which would drive up prices and escalate farm bankruptcies. McDingle is just wrong.

6. Tanya claims that generic drugs are just as effective as their brand name equivalents. Of course, she’d say that. A lot of the funding for her research comes from generic drug companies. I’m going to stick with the brand names.

7. During her interview for the Donnybrook Fellowship, Francine was asked “Are you still selling black market azaleas?” She denied she was. As a result of her confession to having previously participated in the illegal flower trade, her application has been rejected.

8. All of the components of the table you just assembled are 10 years old. As a result, the table itself is 10 years old as well.

9. No one has ever decisively proved that the Flying Spaghetti Monster does not exist. The only conclusion is that it does.

10. I think you’d have to agree that this bicycle now belongs to me. Biff, Spike, and I would be very disappointed if there were any disagreement on this point. Very disappointed.

11. Either it was Beyoncé I saw making sandwiches in the Marquis Culinary Centre or I am blind as a bat. But I get my eyes tested regularly and there are never any problems. It was definitely her.

12. According to the US Constitution, American citizens have a right to bear arms. Hence, any attempt to confiscate my thermonuclear bomb would be completely illegal.

13. Since s/he who laughs last laughs longest, and s/he who laughs longest burns the most energy while laughing, it follows s/he who laughs last burns the most energy while laughing.

14. Driving while high is perfectly safe. People who use marijuana are simply far less likely to get into a car after a night on the town – or get in fights, or engage in vandalism, or anything – than people who have been drinking. The worst thing potheads are going to do is clean out the chip aisle at the convenience store.

15. Trevor McGee claims that a vegetarian diet is both healthier and more humane. But I caught him eating a Big Mac yesterday. I think we can safely disregard his views.

16. Vito allegedly assaulted some guy with a baseball hat. But even if that's true it follows that he couldn't have really hurt the guy.

17. The spread of misinformation through social media allows foreign powers to interfere in our elections, exacerbates health crises, and poisons public discourse. The conclusion is clear: we need to shut down the internet.

18. Drake is Canadian and a high-profile musician. So too are Celine Dion and Justin Bieber. It follows that all Canadians are high-profile musicians.

19. Jeff says that the Stooges are an excellent band. His view seems to be that we should all be forced to listen to the sounds of fingernails scraping across a chalkboard. But nobody should be forced to listen to that.

20. My dad is really mad ever since I crashed his car. The only thing to do is to have him institutionalized.

21. If I get a grade lower than 90 in your course, Professor Alward, I'll lose my scholarship and my family and all the people who supported me back home will be deeply disappointed in me. I really deserve a 90.

22. If I had been running a grades-for-cash scheme, I would have made millions dollars by now. And if I had millions of dollars, I would have retired to a tropical island and opened a tiki bar by now. And if I had retired to a tropical island, I would be basking on the beach wearing a Hawaiian shirt right now. But as you can see, I am currently shivering on the street in Saskatoon wearing a parka. There's no grades-for-cash scheme.

23. Either I will get my promotion or we live in a dark and cruel universe, entirely devoid of goodness or justice. But I see people performing acts of kindness every day. The conclusion is clear.

24. Marcie claims that green energy is going to take off. She would. She's looking for investors for her new solar power company.

25. Saskatoon has a population of nearly 300, 000 people. It follows that each neighborhood in Saskatoon has a population of nearly 300, 000 people.

26. It is simply impossible to prove the nonexistence of anything. It follows that everything exists.

27. All true Husky's fans wear U of S apparel. Join "The Pack."

28. According to the menu, Rashid can either have a salad with his burger or he can have fries. Rashid hates salads. It follows that he will have fries with his burger.

29. Speed kills. As a result, the Police need to start ticketing ambulance drivers who exceed the posted speed limit.

30. Remote teaching has resulted in a disconnect between instructors and their students. Learning outcomes are substantially lower. And the rates of student academic misconduct have soared. For these reasons, the University of Saskatchewan needs to get rid of learning management systems, such as Canvas, altogether and return to old-fashioned paper-and-pencil learning.

Chapter 4: Categorical Propositions

1. The Components of Categorical Propositions

The focus of this chapter is on certain kinds of propositions – categorical propositions. It is worth noting from the beginning that in this discussion the terms “proposition” and “statement’ are used interchangeably. Recall from Chapter 1 that a statement is sentence that is used to make a claim about how things are and, as such, is either true or false. As a result, propositions should be understood in the same way. Some, but not all, propositions are categorical propositions. A **categorical proposition** is a proposition that relates two classes of things. In particular, it is a proposition that claims that a certain number of the members of one class are included in or excluded from another class. Examples of categorical propositions include the following:

High school teachers are undervalued professionals.

Notice that there are two classes under consideration in this proposition: high school teachers, and undervalued professionals. In addition, the proposition makes a claim about the relation between the members of these two classes, namely that all members of the class of high school teachers fall within the class of undervalued professionals.

As a first step to analyzing categorical propositions, we need to introduce the notion of a term. A **term** is a word or phrase that can serve as the subject of a proposition. Terms include proper names, such as “Joni Mitchell” and “Saskatoon,” common names, such as “dog” and “student,” and descriptive phrases, such as “short spy” and “vermin in my kitchen.” A categorical proposition contains two terms, each of which denotes one of the two classes that are related by the proposition: a subject term and a predicate term. The **subject term** denotes the class whose members are claimed to be included in or excluded from a class of things by the categorical proposition; the **predicate term,** in contrast, denotes the class of things that members of the subject class are claimed to be included in or excluded from. Consider, for example, the following categorical proposition:

Some dogs are not good pets.

In this proposition it is the class of dogs that are claimed to fall outside a class of things, and it is the class of good pets that they are claimed to fall outside of. As a result, the expression “dogs” is the subject term and the expression “good pets” is the predicate term.

There are two distinctions in terms of which categorical propositions can be themselves categorized. First, they can be distinguished in terms of whether they make claims about all members of the class denoted by the subject term or only some members of that class. And second, they can be distinguished in terms of whether they claim members of the class denoted by the subject term are included in or excluded from the class denoted by the predicate term. These two distinctions yield exactly four types of categorical propositions:

1. Propositions which claim all members of the subject class are included in the predicate class

2. Propositions which claim that all members of the subject class are excluded from the predicate class

3. Propositions which claim that some members of the subject class are included in the predicate class

4. Propositions which claim that some members of the subject class are excluded from the predicate class.

A **standard form** categorical proposition is a categorical proposition that expresses one of these relations with complete clarity. In order to be in standard form, a proposition has to be an instance of one of the following forms, with "S" and "P" replaced by a subject term and a predicate term respectively:

1. All S are P

2. No S are P

3. Some S are P

4. Some S are not P

Consider, for example, the categorical proposition

No professors are stinging insects.

Because this proposition can be generated by replacing "S" with "professors" and "P" with "stinging insects" in "No S are P," it counts as a standard form categorical proposition. But because the categorical proposition

Not all alien invasions are unpleasant experiences

cannot be generated from any of 1-4 by replacing "S" and "P" with "alien invasions" and "unpleasant experiences," it does not count as a standard form categorical proposition.

In addition to subject and predicate terms, categorical propositions contain quantifiers and copulas. A **quantifier** is an expression which specifies how many members of the subject class are (claimed to be) included in or excluded from the predicate class. In standard form categorical propositions there are exactly three possible quantifiers: "all," "no," and "some." A **copula** is an expression which links the subject term with the predicate term. In standard form categorical propositions there are exactly two possible copulas: “are” and “are not.” Consider the following standard form categorical proposition:

> Some people at the University of Saskatchewan are not things that make good paperweights.

It can be exhaustively analyzed in terms of the four types of expressions we have mentioned as follows:

> *Quantifier*: some
>
> *Subject term*: people at the University of Saskatchewan
>
> *Copula*: are not
>
> *Predicate term*: things that make good paperweights.

When you encounter a standard form categorical proposition, it is always a good idea to underline the quantifier and the copula in order to make the structure of the proposition obvious. In the example under consideration, this would look as follows:

> <u>Some</u> people at the University of Saskatchewan <u>are not</u> things that make good paperweights.

Doing this as a matter of course will make some of the techniques introduced subsequently much easier to learn.

Exercises Section 4.1

Identify the quantifier, the copula, the subject term, and the predicate term in each of the following categorical propositions.

1. All things that go bump in the night are monsters hiding in my closet.

2. Some very important people are not purveyors of depth or substance

3. Some classes taught at U of S are hotbeds of pestilence and disease.

4. No objects I have in my pocket are invisibility rings.

5. Some members of outlaw motorcycle gangs are not people I would bring home to meet my parents.

6. All spicy curries are meals you should not serve to toddlers.

7. Some people who are not worthy of a Nobel Prize are people who are worthy of less prestigious prizes.

8. No socks without holes in them are clothes in need of darning.

9. All people who drink too much and talk to loud are inappropriate tearoom patrons.

10. Some questions on the multiple-choice section of the final exam are not questions that are entirely unfair.

2. Types of Categorical Propositions

A. Quality and Quantity

Two important features of categorical propositions are their quality and their quantity. The **quality** of a categorical proposition is a matter of whether it affirms or denies class membership, that is, whether it claims that members of the class denoted by the subject term are included in or excluded from the class denoted by the predicate term. **Positive propositions** are categorical propositions which affirm class membership, that is, which claim that members of the subject class fall within the predicate class. The categorical proposition

Some donkeys are cooperative pack animals

is a positive proposition because it claims that members of the class of donkeys fall within the class cooperative pack animals. **Negative propositions**, in contrast, are categorical propositions which deny class membership, that is, which claim that members of the subject class are excluded from the predicate class. The categorical proposition

No stained shirts are appropriate fine dining garments

is a negative proposition because it denies that members of the class of stained shirts fall within the class of appropriate fine dining garments.

The **quantity** of a categorical proposition depends on whether it makes a claim about every member of the class denoted by the subject term or just some members of that class. **Universal propositions** are categorical propositions which make claims about every member of the subject class. The proposition

No stained shirts are appropriate fine dining garments

is a universal proposition because it claims that every member of the class of stained shirts is excluded from the class of fine dining garments. **Particular propositions**, in contrast, are categorical propositions which make claims about one or more members of the subject class, but not all members of that class. The proposition

Some donkeys are cooperative pack animals

is a particular proposition because it claims that one or more members of the class of donkeys falls within the class of cooperative pack animals, but it does not claim that every member of the class of donkeys so falls.

B. Types of Propositions

Using the features of quantity and quality, we can distinguish between four categories of propositions: A propositions, E propositions, I propositions, and O propositions. **A propositions** are universal positive propositions: they claim that all members of the class denoted by the subject term are included in the class denoted by the predicate term. Every (standard form) A proposition is an instance of the form

All S are P.

E propositions are universal negative propositions: they claim that all members of the subject class are excluded from the predicate class. Every (standard form) E proposition is an instance of the form

No S are P.

I propositions are particular positive propositions: they claim that one or more members of the subject class are included in the predicate class. Every (standard form) I proposition is an instance of the form

Some S are P.

Finally, **O propositions** are particular negative propositions: they claim that one or more members of the subject class are excluded from the predicate class. Every (standard form) O proposition is an instance of the form

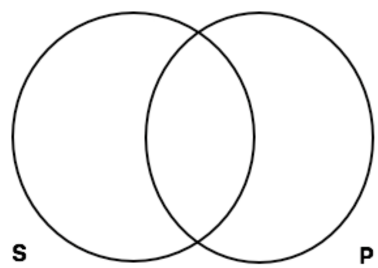
Some S are not P.

This information is compiled in the table below.

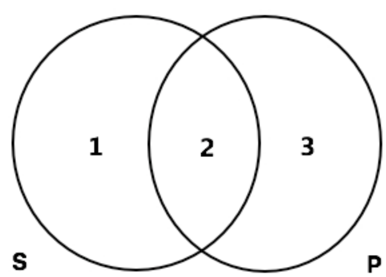
Letter Name	**Form**	**Quality**	**Quantity**
A	All S are P	Positive	Universal
E	No S are P	Negative	Universal
I	Some S are P	Positive	Particular
O	Some S are not P	Negative	Particular

C. Venn Diagrams

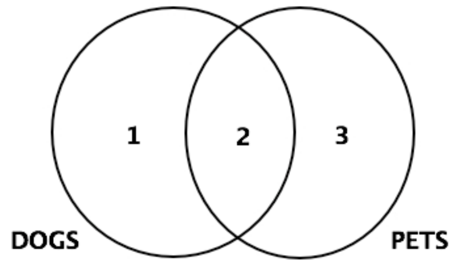
The information expressed by categorical propositions can be represented using Venn diagrams. A **Venn diagram** is an arrangement of overlapping circles each of which represents one of the classes denoted by the terms in a categorical proposition. For convenience, we will adopt the convention of using the left-hand circle to represent the class denoted by the subject term in a categorical syllogism and the right-hand circle to represent the class denoted by the predicate term.



Within the interlocking circles there are three distinct regions, which can be numbered as follows:



Region 1 represents those things which fall inside the subject class but outside the predicate class; region 2 represents those things that fall inside both the subject class and the predicate class; and region 3 represents those things that fall outside the subject class but inside the predicate class. Suppose, for example, that the subject term is “dogs” and the predicate term is “pets,” as per the following Venn diagram:



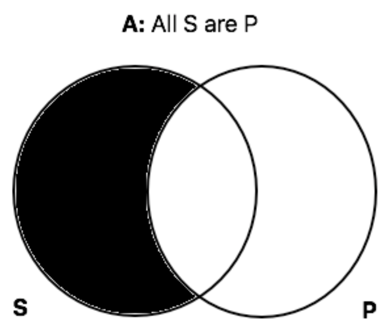
In this case, region 1 would represent dogs that are not pets, region 2 would represent dogs that are pets, and region 3 would represent pets that are not dogs (such as cats).

In order to represent the information expressed by categorical propositions using Venn diagrams, we need to adopt two marking conventions:

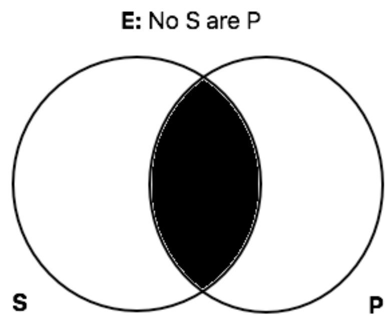
Convention 1: shading a region means that it is empty

Convention 2: placing an X in a region means it contains at least one thing

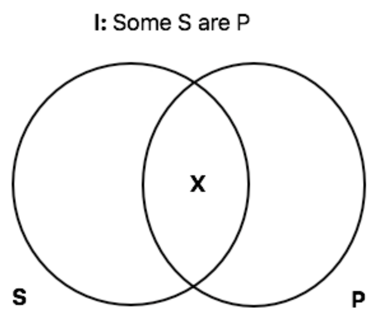
A propositions – propositions of the form “All S are P” – get represented as follows:



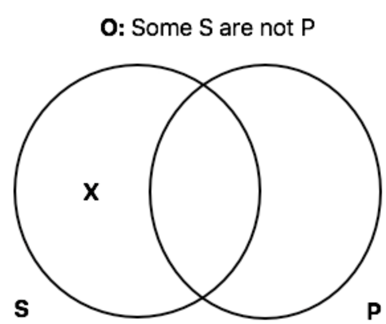
Since in this diagram region 1 is empty, this means that all members of the subject class must fall in region 2; this, of course, means that they all must fall in the predicate class. E propositions – propositions of the form “No S are P” – get represented as follows:



Since in this case region 2 is empty, this means that all members of the subject class must fall in region 1; this, of course, means that none of them can fall in the predicate class. I propositions – propositions of the form "Some S are P" – get represented as



In this case, the X in region 2 means that there is at least one thing in that region; and since that region represents things that are both in the subject class and the predicate class, this means that at least one member of the subject class is also a member of the predicate class. Finally, O propositions – propositions of the form "Some S are not P" – get represented as



In this case, the X in region 1 means that there is at least one thing in that region; and since that region represents things that are inside the subject class but outside the predicate class, this means that at least one member of the subject class is not a member of the predicate class.

Exercises Section 4.2

Indicate the ***quality*** *of each of the following categorical propositions*.

1. Some people with chips on their shoulder are people who shouldn't be in charge of the dip.

2. No drunken masters are people who win martial arts competitions,

3. All people who refuse to socially distance are people who don't care about the well-being of others.

4. Some waffle cone aficionados are not fans of soft ice cream.

5. No people of high moral caliber are fans of X-rated movies.

Indicate the ***quantity*** *of each of the following categorical propositions*.

6. No places with effective air conditioning are places the homeless are allowed to linger.

7. Some pizzas that are unpleasant when served hot are not pizzas that are unpleasant when served cold.

8. Some endangered species of insects are animals that pose a risk to human health.

9. All objects that can be found in my pockets are things that are best not shared with menacing cave dwellers.

10. No shoes that I would consider purchasing are footwear with wheels or wings.

3. Translations into Categorical Form

Relatively few statements one might encounter are standard form categorical propositions. Despite the clarity and precision that such propositions have, people for the most part just do not talk and write that way. Nevertheless, in many cases it is possible to translate such statements into standard categorical form. And not only does doing so clarify exactly what claims are being made by the statements in question, many of the techniques we will be developing can only be applied to standard form categorical propositions. As a result, translating statements into categorical form can significantly increase the range of cases in which these techniques can be deployed.

Your goal in translating a statement is to produce a standard form categorical proposition which means the same as the original. This involves deciding what classes of things are at issue in the original, choosing terms which denote those classes, determining which of the four categorical forms best captures the meaning of the original, and deciding which of the terms you choose is the subject term and which is the predicate term. Because the meanings of standard form categorical propositions are often more precise than those of the statements being translated, in many cases you will not end up with a proposition that means exactly the same as the original. As a result, you will often have to make judgment calls about which standard form propositions best capture their meanings. In what follows, we will develop a number of "rules of thumb" for helping you do just that.

A. The Basics – Terms, Copulas, and Quantifiers

In some cases, translating statements in standard categorical just requires a little bit of tidying up. Sometimes the expressions in the subject or predicate position of a target statement are not terms – nouns or noun phrases, such as "ducks' or "socks that have not been washed in a year," which denote classes of things. Sometimes the subject and predicate terms are linked by verbs other than the standard form copulas "are" and "are not." And sometimes a target statement includes a quantifier other than the standard form "all," "no," or "some," or even no quantifier at all. We will consider each of these issues in turn.

First, consider a statement which has an adjective in the predicate position rather than a term, such as the following:

All professors are evil.

The word "evil" does not denote a class of things but rather picks out a property that things may possess. One can, however, easily transform some such statement into a standard form proposition by combining the adjective with a plural noun, such as "people' or "things," as follows:

<u>All</u> professors <u>are</u> evil people.

There will always be choices about how to transform an adjective into a proper predicate term, such as the following:

<u>All</u> professors <u>are</u> evil things.

<u>All</u> professors <u>are</u> evildoers.

It normally does not matter which expression you choose as long as you do so consistently.

Second, consider a statement in which the subject and predicate terms are linked by a verb other than "are" or "are not," such as,

No Nova Scotians dislike seafood chowder.

This statement can be translated into standard form by inserting one of the two copulas after the subject term and transforming the remainder of the statement into a noun phrase, yielding the following:

<u>No</u> Nova Scotians <u>are</u> people who dislike seafood chowder

Third, consider statements which have nonstandard quantifiers, such as "a few" and "not all." Examples include,

A few AC/DC albums are records in my collection

Not all novelists are destined for disappointment.

Statements with the quantifier "a few" are normally translated as I statements, as in

<u>Some</u> AC/DC albums <u>are</u> records in my collection.

And statements with the quantifier "Not all" (or "not every") are normally translated as O statements, as in

<u>Some</u> novelists <u>are not</u> destined for disappointment.

Next consider statements that lack quantifiers altogether, such as,

Sisters are female siblings

Mice live under the sink in my kitchen.

In deciding how to translate such statements, you need to figure out what their most plausible meaning is. The main thing you need to decide is whether the quantity of the statement is universal or particular, that is, whether the statement is best understood to be making a claim about all members of the subject class or just some members of it. Given the definition of the word "sister," the first statement is more plausibly translated as

<u>All</u> sisters <u>are</u> female siblings

than as

<u>Some</u> sisters <u>are</u> female siblings.

Similarly, given the numbers of mice in existence and the limited space under any given kitchen sink, the second statement is more plausibly translated as

Some mice are things that live under my kitchen sink

than as

All mice are things that live under my kitchen sink.

Finally, consider cases that combine the standard form quantifier "all" with the standard form copula "are not," such as,

All cats are not canines

All professors are not hard graders.

Such statements are not standard form categorical propositions but can be translated as either E Statements – "No S are P" – or O statements – "Some S are not P." In order to determine which is correct, you need to decide which is more plausible. Given that cats are widely known to be felines rather than canines,

No cats are canines

is a more plausible translation of the first statement than

Some cats are not canines.

And given the range of grading practices among different professors,

Some professors are not hard graders

is a more plausible translation of the second statement than

No professors are hard graders

B. Singular Propositions

Some statements contain terms that denote individual people, places, or things rather than classes of the same. Examples include the following:

Hilary Clinton is an American citizen.

North Battleford is a city in Saskatchewan.

The common room computer is an iMac.

In order to translate singular propositions into standard form categorical propositions, we need to transform the singular terms – names and descriptive phrases which denote individual things – into expressions which denote classes. But in order to end up with a proposition which means the same as the original statement, the only member of the class denoted by the new expression can be the individual thing denoted by the original term. This result can be achieved by using expressions such as

people identical to

places identical to

things identical to,

as follows:

All people identical to Hilary Clinton are American citizens

All places identical to North Battleford are cities in Saskatchewan

All things identical to the common room computer are iMacs

It is worth noting that the sense of identity here is numerical identity – being the one and the same person/ place/ things as – rather than exact similarity. After all, while many people might be exactly similar to Hilary Clinton, only Hilary Clinton herself is one and the same person as Hilary Clinton.

C. Times and Places

Instead of the standard form quantifiers "all," "no," and "some," some statements contain spatial adverbs, such as "where," "wherever," "anywhere," "everywhere," and "nowhere," while others contain temporal adverbs such as "when," "whenever," "anytime," "always," and "never." Examples include the following:

Birds can be found where feeders are well-maintained.

Basmati rice is nowhere to be found in Sobeys

Whenever it rains it pours

Mary never eats Brussel sprouts

In such cases, the best translations involve taking the subject and predicate terms to denote classes of places and times respectively, rather than classes of things denoted by other noun phrases in the original statements. As a result, we might translate the above statements as follows:

All places feeders are well-maintained are places birds can be found

No places in Sobeys are places Basmati rice can be found

All times it rains are times it pours

No times are times Mary eats Brussel sprouts

D. Conditional Statements

Conditional Statements are statements of the form

If P then Q

where “P” and “Q” are both propositions. The proposition “P” – which occurs between “if” and “then” – is the antecedent of the conditional and the proposition “Q” – which occurs after “then” – is the consequent. Conditional statements can usually be translated into A statements. In order to do so, you need to transform the antecedent of the conditional into the subject term of the categorical proposition and the consequent into the predicate term. Consider the following conditional statements:

If it is snowing, then it is cold

If a professor shouts her lectures then s/he makes the students’ ears bleed

These can each be translated into standard form categorical propositions as follows:

All places it is snowing are places it is cold.

All professors who shout their lectures are professors who make their students’ ears bleed.

It is worth noting, however, that if the consequent of a conditional is negated, it is sometimes more natural to translate it as an E statement. So while

If a professor whispers her lectures, then she does not make her students’ ears bleed

could be translated as

All professors who whisper their lectures are professors who do not make their students’ ears bleed,

it might more naturally be translated as

No professors who whisper their lectures are professors who make their students' ears bleed.

E. "Only" vs. "The only"

Many propositions that involve the expressions "only" or "the only" can be translated into standard form as A statements. Examples include the following:

The only mammals are animals.

Only mammals are dogs.

Doctors are the only surgeons.

Fred's blacklist includes only surgeons.

The challenge with translating statements of these kinds into standard form is determining which expression is the subject term and which expression is the predicate. The following rule of thumb is helpful: an expression that immediately follows "the only" is the subject term in an A statement translation, whereas an expression that follows "only" by itself (without the definite article "the") is the predicate term. Applying this rule of thumb yields the following translations of our examples:

All mammals are animals.

All dogs are mammals.

All surgeons are doctors.

All people on Fred's blacklist are surgeons.

Exercises Section 4.3

Translate each of the following statements into standard categorical form.

1. Not all seven-footers are good basketball players.

2. Some rescue dogs are unhappy in their new homes.

3. Good live music is nowhere to be found in Regina.

4. All piano teachers eventually lose interest in the instrument.

5. Only cowboys sing the blues.

6. Squirrels with red fur and bushy tails are nesting in the crawl space in behind my closet.

7. If a tree falls alone in the forest, then it does not make a sound.

8. All sparrows are not mammals.

9. The only thing that makes me laugh these days is a politician self-destructing in public.

10. Ottawa is the capital city of Canada.

11. You should brush your teeth if you have a late-night snack.

12. No serial plagiarizers are well-liked by their professors.

13. Victims of the defendant’s con only include very naïve and lonely people.

14. A few outlaw motorcycle club members get extremely high scores on the LSAT.

15. If you eat everything on your plate, then you won’t have to clean up the dinner dishes.

16. Some chefs in fancy restaurants don’t like the food they prepare for their customers.

17. Dr. Alward is not a very nice person.

18. Not all people with poor personal hygiene and filthy clothing consider themselves to be undesirable romantic partners.

19. People who don’t want to get in trouble with the law are the only people who use the anonymous tip line.

20. Dr. Alward arrives to save the day whenever egregious logical fallacies are committed.

4. Rules of Immediate Inference

A. Transforming Categorical Propositions

Consider the following pairs of categorical propositions:

All ducks are swimmers	No Ducks are non-swimmers

Some dogs are good pets   Some good pets are dogs

Although in each case we have distinct propositions, the members of each pair have the same meaning and, as a result, must have the same truth-value; that is, if one is true, the other must be true as well, and if one is false, the other must be false as well. In order to determine whether two categorical propositions are related in this way we need to determine (1) by means of what operations the members of the pair can be transformed into one another and (2) whether the application of an operation to a given target proposition yields an equivalent proposition. In this section, we will consider question (1). We will take up (2) in the next section.

The three operations we will be utilizing here – conversion, obversion, and contraposition – each consist of the combination of one or more of the following three transformations: (a) switching the subject and predicate terms; (b) changing the quality of the proposition; (c) replacing one or more terms with their complements. As you may recall, a categorical proposition consists of four expressions: a quantifier, a subject term, a copula, and a predicate term. In the statement,

Some iguanas are not excellent singers,

the quantifier is "some," the copula is "are not," the subject term is "iguanas," and the predicate term is "excellent singers." It will prove helpful in what follows to underline the quantifier and the copula, as I have done above, in order to keep track of which expressions need to be altered and which do not.

Transformation (a) – switching the subject and predicate terms – is as straightforward as it seems. It simply involves moving the term which is initially in the subject position – between the quantifier and the copula – to the predicate position – after the copula – and moving the term initially in the predicate position to the subject position. In our example, since "iguanas" is initially in the subject position, it needs to be moved to the position after the copula; and since "excellent singers" is initially in the predicate position, it needs to be moved to the position between the quantifier and the copula. This yields the following proposition:

Some excellent singers are not iguanas

There are two things one needs to be careful of when switching the subject and predicate: making sure you move the whole subject and the whole predicate; and making sure you do not change either the quantifier or the copula. The proposition,

Some singers are not excellent iguanas,

would not count as the product of transformation (a) because only part of the term originally in the position – "excellent singers" – has been moved to the predicate position. And the proposition

Some excellent singers are iguanas

would not count as the product of transformation (a) because the copula has been changed from "are not" to "are."

Transformation (b) – changing the quality of the proposition – is slightly more complicated than (a) but is still quite straightforward. As you may recall, the **quality** of a categorical proposition is a matter of whether it affirms or denies class membership, that is, whether it claims that members of the class denoted by the subject term are included in or excluded from the class denoted by the predicate term: if it affirms class membership then its quality is positive; if it denies class membership then its quality is negative. In order to change the quality of a categorical proposition, you need to change either the quantifier or the copula, but not both; and which you need to change depends on the type of categorical proposition the initial proposition is. If the initial proposition is an A-proposition – of the form "All S are P" – or an E-proposition – of the form "No S are P" then you need to change the quantifier. In particular, if the quantifier is "all" it gets replaced with "no," and if it is "no" it gets replaced with "all." And if the initial proposition is an E-proposition – of the form "Some S are P" – or an O-proposition – of the form "Some S are not P" – then you need to change the copula. In particular, if the copula is "are" it gets replaced with "are not," and if it is "are not" it gets replaced with "are." This information is compiled in the table below.

Initial Proposition	**Transformed Proposition**
A: All S are P	No S are P
E: No S are P	All S are P
I: Some S are P	Some S are not P
O: Some S are not P	Some S are P

Suppose, again, that our initial proposition is

Some iguanas are not excellent singers

Given that this is an O-proposition, changing the quality yields the corresponding I-proposition

Some iguanas are excellent singers.

Transformation (c) – replacing one or more terms with their complements – is perhaps the most difficult of the three, but again it is still fairly easy to understand. The first thing to remember is that terms denote classes of things. The term “iguanas,” for example, denotes a certain class of lizards. The **complement** of a term is an expression which denotes the class whose members consist of everything that falls outside the class denoted by the original term. So, for example, the complement of “iguanas” is “non-iguanas” or “things that are not iguanas.” There is usually more than one way to form the complement of a term. If the term consists of a single word, one can, as above, always prefix “non” to it. And if the term already contains a prefix, you can sometimes form the complement by removing it. For example, a term complement of “immortals” is “mortals.” But if you are not sure what to do, you can always put the expression “things that are not” immediately in front of the initial term. Suppose that you are asked to replace the predicate term with its complement in

Some iguanas are excellent singers.

One way of doing so would yield the following:

Some iguanas are *things that are not* excellent singers.

Let us turn now to the three operations that are the central focus of this section: conversion, obversion, and contraposition. Conversion is the simplest of the three. **Conversion** simply consists in subjecting the initial proposition to transformation (a), that is, switching the subject and predicate terms in that proposition. The proposition that results from this operation is the **converse** of the initial proposition. So, for example, the converse of

All macadamia nuts are items banned in the classroom

is

All items banned in the classroom are macadamia nuts.

Unlike conversion, obversion requires the use of two transformations and not one. In particular, **obversion** consists in subjecting the initial proposition to transformation (b) – changing the quality of the proposition – and transformation (c) – replacing the predicate term with its complement. The proposition that results from this operation is the **obverse** of the initial proposition. Suppose our initial proposition is, again,

All macadamia nuts are items banned in the classroom.

At the first stage in finding the obverse, we change the quality of this proposition. Since it is an A-proposition, this yields,

No macadamia nuts are items banned in the classroom.

At the second stage, we replace the predicate term "items banned in the classroom" with its complement. One term that would suffice is "items permitted in the classroom." Using this as the complement term yields the following,

> No macadamia nuts are items permitted in the classroom,

which therefore counts as the obverse of the original proposition.

Finally, contraposition, like obversion, also requires the use of two transformations. In particular, **contraposition** consists in subjecting the initial proposition to transformation (a) – switching the subject and predicate terms – and transformation (c) – replacing both the subject term and the predicate term with their complements. The proposition that results from this operation is the **contrapositive** of the initial proposition. Suppose our initial proposition is once more,

> All macadamia nuts are items banned in the classroom.

At the first stage in finding the contrapositive, we switch the subject and predicate terms, which yields,

> All items banned in the classroom are macadamia nuts.

At the second stage, we replace both the initial subject term, "macadamia nuts," and the initial predicate term, "items banned in the classroom," with their complements. For illustrative purposes I will use "things that are not macadamia nuts" and "items permitted in the classroom" respectively. This yields

> All items permitted in the classroom are things that are not macadamia nuts

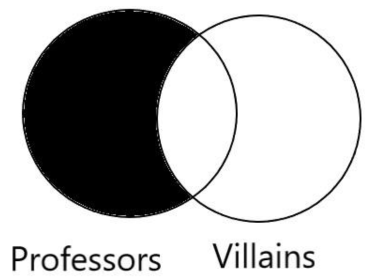
as the contrapositive of the initial proposition.

B. Identifying Equivalences

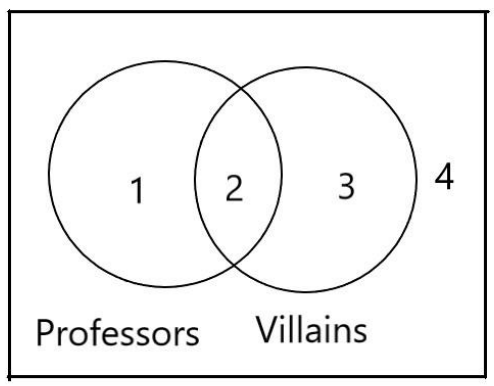
The central goal of this module is not just to be able to transform categorical propositions in various ways but rather to be able to determine whether they are logically equivalent to one another. To say a pair of statements are **logically equivalent statements** is to say they have the same meaning and, therefore, that they necessarily have the same truth-value. The meaning of a categorical proposition is given by its Venn diagram. Hence, two propositions have the same meaning just in case they have the same diagram. Consider, for example, the following pair of propositions:

> (a) All professors are villains (b) No professors are non-villains

As we have seen, the diagram for (a) is as follows:



In order to determine whether (b) has the same diagram as (a) we need to use the following numbering,



where region 4 corresponds to the class of things that are neither professors nor villains. As an E-proposition, (b) claims that the intersection of the subject term and the predicate term is empty. The subject term, “professors,” denotes entities which might fall in either regions 1 or 2. Since the term “villains” denotes entities which might fall in either regions 2 or 3, its complement, “non-villains,” denotes entities that might fall in either regions 1 or 4. As a result, the intersection of “professors” and “non-villains” is region 1. But if that is shaded as (b) claims it should be, then what we end up with is the diagram from (a). Hence, (a) and (b) have the same Venn diagram and so are logically equivalent statements.

You might have noticed that in the previous example, (b) is the obverse of (a). And the result we established concerning (a) and (b) is a general one: every categorical proposition is logically equivalent to its obverse. In the case of the other operations, the equivalences only hold in certain cases. In particular, E-propositions and I- propositions are logically equivalent to their converses, while A-propositions and O-propositions are not; and A-propositions and O-propositions are equivalent to their contrapositives while E-propositions and I-propositions are not. This information is compiled in table OA-4 below.

Proposition	Obverse	Converse	Contrapositive
A: All S are P	Equivalent	Not equivalent	Equivalent
E: No S are P	Equivalent	Equivalent	Not equivalent
I: Some S are P	Equivalent	Equivalent	Not equivalent
O: Some S are not P	Equivalent	Not equivalent	Equivalent

.

Finally, consider the following three propositions:

(i) Some Nova Scotians are not people with funny accents

(ii) Some people with funny accents are not Nova Scotians

(iii) Some people without funny accents are not things that are not Nova Scotians

The first thing to note is that (ii) is the converse of (i) and (iii) is the contrapositive of (i). But since (i) is an O-statement, Table OA-4 tells us that (iii) is logically equivalent to (i) but that (ii) is not equivalent to (i).

Exercises Section 4.4

Find the converse, obverse, and contrapositive of each of the following statements. Indicate whether or not each is equivalent to the original.

1. All people wearing fedoras are stylish dressers.

2. No things that go bump in the night are scary monsters.

3. Some people with foul-smelling feet are ballerinas.

4. Some things I'd rather not say are not secrets.

5. All things which require constant maintenance are things that end up in the landfill.

6. No spices that need to be roasted prior to use are items cluttering up my spice rack.

7. Some crotchety old professors are people who prefer remote teaching.

8. Some new-fangled flavors of gum are not items developed in secret research laboratories.

9. All people who think they can get my goat are people with little experience on the farm.

10. No asinine tales of virtue and heroism are stories permitted to be told in my living room.

11. Some strategies for getting acquaintances to help you move are things which pose the risk of uncomfortable dinner dates.

12. Some people whose views are resistant to contrary evidence are not people who reject conspiracy theories.

Chapter 5: Categorical Syllogisms

1. Features of Categorical Syllogisms

A. Standard-Form Categorical Syllogisms

A **syllogism** is a basic kind of deductive argument (which, recall, is to say that it incorporates the claim that it is impossible for the conclusion to be false given that the premises are true). In particular, it is a deductive argument with exactly two premises and one conclusion. An argument counts as a **categorical syllogism** if the following conditions are met:

- The argument is a syllogism.
- Both premises and the conclusion of the argument are categorical propositions.
- The premises and conclusions contain exactly 3 different terms between them.
- Each term appears twice in different propositions.

Consider the following argument:

All logicians are awkward conversationalists
<u>Some logicians are not tango aficionados</u>
Some awkward conversationalists are not tango aficionados

Because it contains two premises and a conclusion, this argument counts as a syllogism. And each of the premises and the conclusion are categorical propositions, making claims about the relations between classes. Jointly they contain three terms: "logicians," "awkward conversationalists," and "tango aficionados." Moreover, "logicians" occurs in each of the premises, "awkward conversationalists" occurs in the first premise and the conclusion, and "tango aficionados" occurs in the second premise and the conclusion. As a result, since the argument meets all of the requirements listed above, it counts as a categorical syllogism.

Moreover, each of the three terms has a designation based on where it occurs in the argument. The **major term** is the term that occurs as the predicate of the conclusion and in one of the premises. In the example above, this would make "tango aficionados" the major term. The **minor term** is a term that occurs as the subject of the conclusion and in one of the premises. "Awkward conversationalists" is the minor term in our example. Finally, the **middle term** is the term that occurs in both premises but does not occur anywhere in the conclusion. This would make "logicians" the middle term in our example. It is also convenient to label the premises of a categorical syllogism in a way that reflects which of these terms occur in them. The **major premise** of a categorical syllogism is the premise in which the major term occurs. In our example this would be

Some logicians are not tango aficionados.

The **minor premise**, in contrast, is the premise in which the minor term occurs. The statement

All logicians are awkward conversationalists

is the minor premise in our example.

It is often convenient to put categorical syllogisms into standard form. A **standard-form categorical syllogism** is one in which the premises and conclusion are listed from top to bottom as follows:

- The major premise is listed first
- The minor premise is listed second
- The conclusion is listed last

When putting a categorical syllogism into standard form, it is important to make sure the following conditions are met:

- Both premises and the conclusion are standard-form categorical propositions
- The two occurrences of each term are the same
- Each term has the same meaning in each of its occurrences

Because the minor premise

All logicians are awkward conversationalists

is listed before the major premise

Some logicians are not tango aficionados

in our example, it is not currently in standard form. This can be easily remedied by reordering the premises as follows:

Some logicians are not tango aficionados [Major Premise]
All logicians are awkward conversationalists [Minor Premise]
Some awkward conversationalists are not tango aficionados

B. Mood and Figure

Once a categorical syllogism has been put into standard-form, its mood and figure can be easily identified. The **mood** of a categorical proposition consists of the letter names of its constituent propositions in the following order: major premise, minor premise, conclusion. Recall: the letter names of the four standard form categorical propositions are as follows:

A: All S are P

E: No S are P

I: Some S are P

O: Some S are not P

Consider, for example, the following standard-form categorical proposition:

Some donkeys are not ill-tempered beasts
<u>No ill-tempered beasts are literate readers</u>
Some literate readers are donkeys

Given that the major proposition is an O-proposition, the minor premise is an E-proposition, and the conclusion is an I-proposition, the mood of the categorical syllogism is **OEI**. But suppose the subject and predicate terms were switched as in

Some donkeys are not ill-tempered beasts
<u>No ill-tempered beasts are literate readers</u>
Some donkeys are literate readers

One might think that the mood was still **OEI**. But the argument is not in standard form. After all, the major premise – in which the major term "literate readers" occurs – is listed second and not first. As a result, the mood is actually **EOI**.

The **figure** of a categorical syllogism is determined by the location of the middle term in the major and minor premises, that is, whether it occupies the subject or predicate position in those premises. Given that the middle term can occupy either of two positions in two premises, there are total of four possible figures a standard-form categorical proposition might have.

Figure 1: the middle term occupies the subject position in the major premise and the predicate position in the minor premise.

M P
<u>S **M**</u>
S P

Figure 2: the middle term occupies the predicate position in both premises

P **M**
<u>S **M**</u>
S P

Figure 3: the middle term occupies the subject position in both premises

M P
<u>**M** S</u>
S P

Figure 4: the middle term occupies the predicate position in the major premise and the subject position in the minor premise

P **M**
<u>**M** S</u>
S P

As with the mood, the figure of a categorical syllogism is much easier to identify if it is in standard form. Consider, again, the following categorical syllogism:

Some donkeys are not ill-tempered beasts
<u>No ill-tempered beasts are literate readers</u>
Some literate readers are donkeys

The middle term in this argument is “ill-tempered beasts” which we can highlight as follows:

Some donkeys are not **ill-tempered beasts**
<u>No **ill-tempered beasts** are literate readers</u>
Some literate readers are donkeys

Because the middle term is in the predicate position in the major premise and in the subject position in the minor premise, it has figure 4.

From the figure and mood of a categorical syllogism alone one can determine whether or not it is valid. The following table indicates which combinations of figure and mood are valid. If a combination does not appear on the table, it is invalid.

Figure 1	**Figure 2**	**Figure 3**	**Figure 4**
AAA	**AEE**	**AII**	**AEE**
AII	**AOO**	**EIO**	**EIO**
EAE	**EAE**	**IAI**	**IAI**
EIO	**EIO**	**OAO**	

Consider, again, the following categorical syllogism:

Some donkeys are not ill-tempered beasts
<u>No ill-tempered beasts are literate readers</u>
Some literate readers are donkeys

As we have seen, the mood of this argument is **OEI** and it has figure 4. And since mood **OEI** does not occur in the column for figure 4, the argument is invalid.

Exercises Section 5.1

Identify the major term, minor term, middle term, major premise and minor premise for each of the following categorical syllogisms.

1. All dogs with funny names are beloved pets
   Some lords of the household are beloved pets
   No lords of the household are dogs with funny names

2. No people with 70's mustaches are friends of mine
   Some friends of mine are refugees from the 60's
   No refugees from the 60's are people with 70's mustaches

3. Some edible clothes items are things that dissolve in water
   All things that dissolve in water are poor rain garments
   Some poor rain garments are edible clothes items

4. No flying pink elephants are fatigue-induced hallucinations
   All fatigued-induced hallucinations are memories of youth
   Some memories of youth are flying pink elephants

5. Some hare-brained schemes are compelling investment opportunities
   All hare-brained schemes are illegal enterprises
   Some illegal enterprises are compelling investment opportunities.

Identify the mood and figure for each of the following categorical syllogisms. Using the table from section 5.1, determine whether the argument is valid.

6. Some strange happenings are not Bigfoot sightings
   All face-to-face classes are strange happenings
   Some face-to-face classes are not Bigfoot sightings

7. All paperback books are weighty tomes
   All graphic novels are paperback books
   All graphic novels are weighty tomes

8. No things I ate for breakfast are healthy food choices
   All things I ate for breakfast are Snickers bars
   No Snickers bars are healthy food choices

9. Some U of S students are logic enthusiasts
   All U of S students are appreciators of fine wines
   Some appreciators of fine wines are logic enthusiasts

10. No moral norms are rules of etiquette
<u>All game-theoretical strategies are rules of etiquette</u>
No game-theoretical strategies are moral norms

11. All popular movie actors are megalomaniacs
<u>Some megalomaniacs are logic professors</u>
Some logic professors are popular movie actors

12. No alternate realities are worlds in which I exist
<u>Some alternate realities are places I am emperor of all things</u>
Some places I am emperor of all things are not worlds in which I exist

13. All delicious popcorn toppings are butter-based sauces
<u>All heart-unfriendly foods are butter-based sauces</u>
All heart-unfriendly foods are delicious popcorn toppings

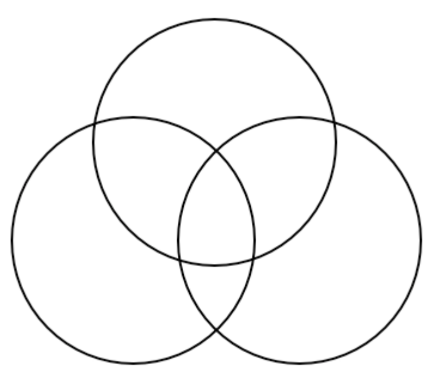
14. All people with unfortunate habits are metaphorical Kool-Aid drinkers
<u>Some corrupt politicians are not people with unfortunate habits</u>
Some corrupt politicians are not metaphorical Kool-Aid drinkers

15. All rotary phones are museum pieces
<u>No museum pieces are items hidden beneath my floorboards</u>
No items hidden beneath my floorboards are rotary phones

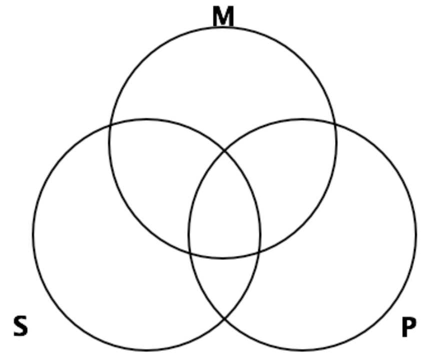
2. Categorical Syllogisms and Venn Diagrams

A. Creating Venn Diagrams

Although we can always assess the validity of a categorical syllogism using by deploying its mood and figure as above, doing so gives little insight into exactly why the arguments being assessed are valid or not. Another technique for assessing validity is the Venn diagram technique, which makes explicit why arguments are assessed in the way that they are. The first step consists in producing a diagram consisting of three interlocking circles.

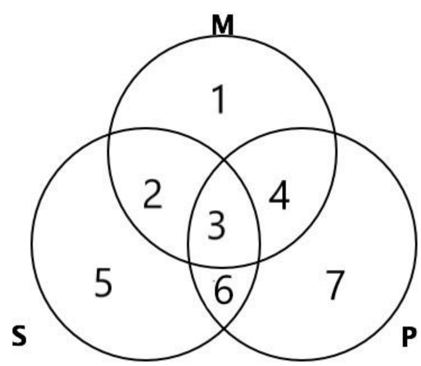


These circles correspond to the classes of things denoted by the three terms that appear in a categorical syllogism. The lower left circle corresponds to the class denoted by the minor term (S), the lower right circle corresponds to the class denoted by the major term (P), and the upper middle circle corresponds to the class denoted by the middle term (M).



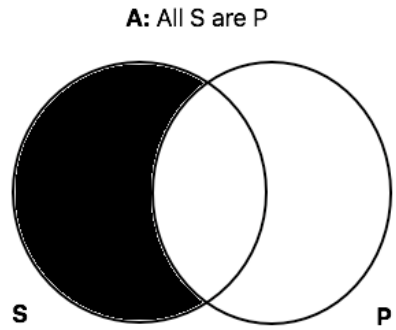
It is important that you label your diagrams in this way. Students who fail to do so will receive a deduction on assignments and tests.

Recall that according to the definition of validity, an argument is valid just in case it is not possible for the premises to be true and the conclusion false. The idea behind the Venn diagram technique is to assume the premises are true and fill in the information they contain on a Venn diagram. Once this is done you assess whether or not the conclusion of the argument could be false given the information you have filled in on it. If the diagram for the premises guarantees that the conclusion is true, then the argument is valid; but if the diagram for the premises leaves open the possibility that the conclusion is false, then the argument is invalid. In this section we will be learning how to represent the information contained in the premises in a Venn diagram; in the next section we will learn how to interpret a completed Venn diagram. For convenience, we will be using the following numbering for both tasks.

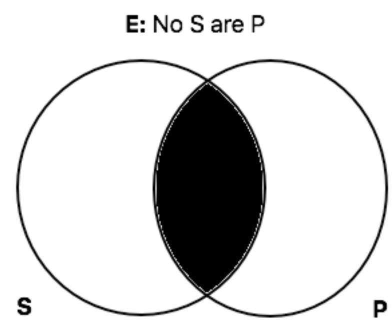


In order to complete a Venn diagram for a categorical syllogism, we will have to make use of the characteristic diagrams for the four standard-form categorical propositions:

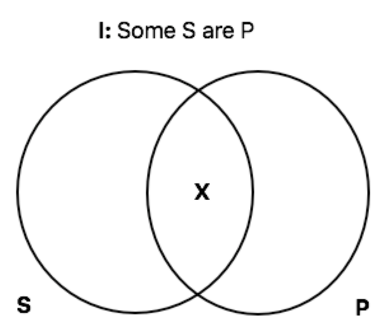
1. A-propositions: All S are P



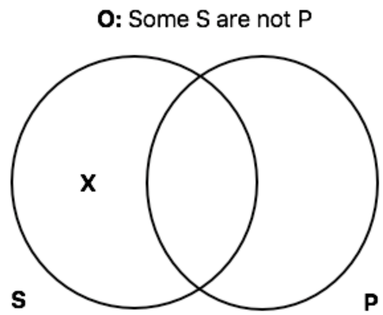
2. E-propositions: No S are P



3. I-propositions: Some S are P



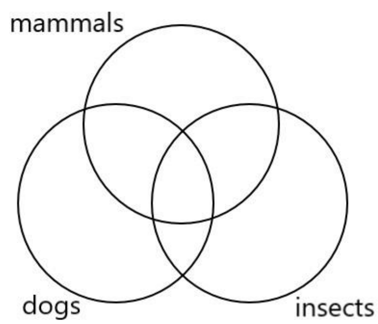
4. O-propositions: Some S are not P



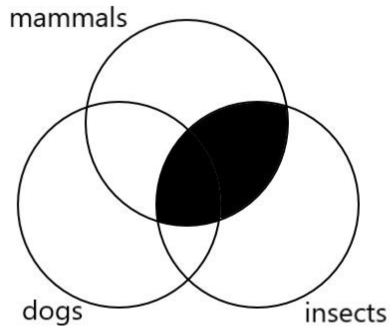
Consider the following categorical syllogism:

No mammals are insects
All dogs are mammals
No dogs are insects

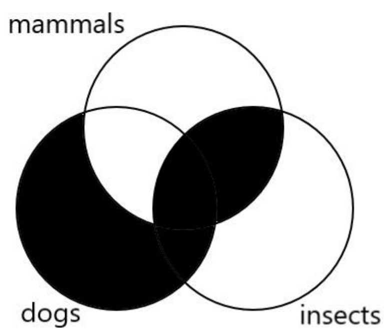
The first thing we do by way of representing this argument with a Venn diagram is label the diagram correctly.



Since both premises are A or E propositions – and, hence, they both require shading regions of the diagram – it does not matter which premise we diagram first. As a result, we will simply diagram the premises in the order in which they occur. The first premise is an E-proposition. According to the characteristic diagram for E-propositions, this means we need to shade in the intersection of the class of mammals and the class of insects. According to our numbering, that requires shading in regions 3 and 4 as follows:



The second premise is an A-proposition. According to the characteristic diagram for A-propositions, this means that we need to shade in the regions of the subject class, “dogs,” that fall outside of the predicate class, “mammals,” namely regions 5, and 6.

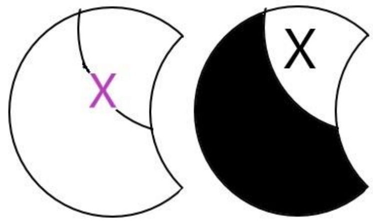


So far, we have looked at an argument whose premises are A- and E-propositions, which require shading regions of Venn diagrams. Arguments with I- and O-propositions – which require putting x’s on diagrams – come with their own set of challenges. These challenges can be met, however, by following two simple rules:

1. If one premise requires shading regions of a Venn diagram and the other requires placing an x on the diagram, *do the shading before placing the x on the diagram*.

2. If an x can go into two separate regions of a Venn diagram, *place it on the line between those regions*.

The point of shading a diagram before putting an x on it becomes clear if one remembers what shading and x’s mean. A shaded region is empty: nothing falls into that category of things. An x in a region means there is at least one thing in it: there is at least one item that falls into that category of things.

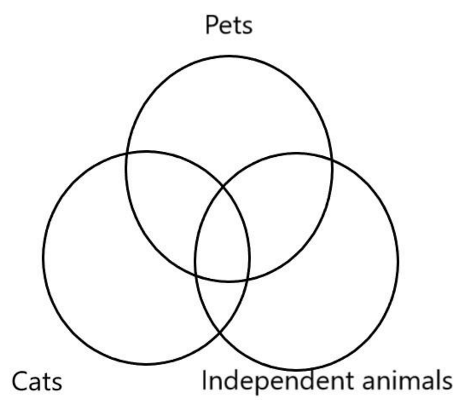


Now suppose that, in the absence of other information, an x could fall into either of two regions. If we know that one of the regions is empty, it follows that it must fall in the other region.

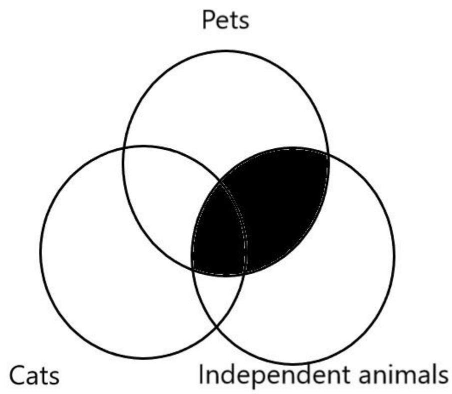
Consider finally a categorical syllogism that requires putting an x on the diagram:

> No pets are independent animals
> <u>Some cats are not pets</u>
> Some cats are independent animals

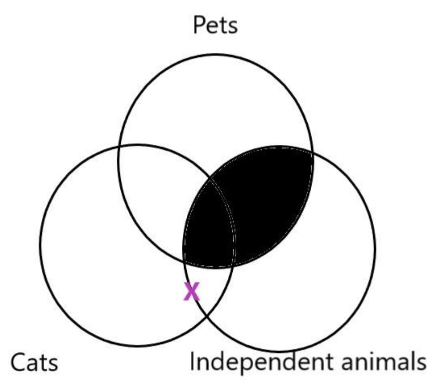
First, we produce a Venn diagram for the argument labelled as follows:



Second, following Rule 1, we diagram the first premise – which requires shading – before we diagram the second – which requires an x. Since the first premise is an E-proposition, this means we need to shade in the intersection of the class of pets and the class of independent animals. According to our numbering, that requires shading in regions 3 and 4 as follows:



Finally, we need to diagram the second premise. Since it is an O-proposition, we need to look at the characteristic diagram for such O-propositions. This tells us we need to put an x on the diagram in a region which falls inside the class of cats but outside the class of pets. Since both regions 5 and 6 are inside the class of cats but outside the class of pets, following Rule 2, we need to put an x on the line between these two regions.

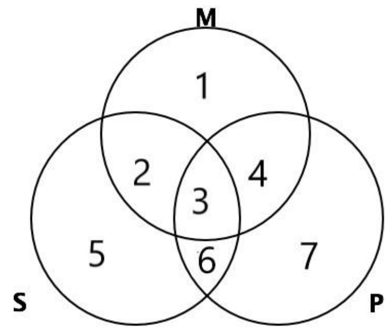


Now that our diagram is complete, our next task is to learn how to interpret it.

B. Interpreting Venn Diagrams

The point of creating Venn diagrams for arguments is to use them to determine whether or not those arguments are valid. Recall: to say that an argument is valid is just to say that it is not possible for the premises to be true and the conclusion false, that the truth of the premises guarantees the truth of the conclusion. And to say that an argument is invalid is to say that the truth of the premises does not guarantee the truth of the conclusion, that even if the premises are true, the conclusion might still be false. In creating a Venn diagram for the premises of a categorical syllogism, we are in effect assuming those premises are true. And the question is whether or not that diagram makes the conclusion true as well. If the diagram from the premises makes the conclusion true, then the argument is valid. After all, if the diagram makes the conclusion true, then it is not possible for the premises to be true and the conclusion false. But, on the other hand, if the diagram for the premises either makes the conclusion false or leaves open the possibility that it is false, then the premises do not make the conclusion true and, hence, the argument is invalid.

In order to determine whether or not a Venn diagram makes the conclusion of an argument true, you only need to know what type of proposition the conclusion is, as well as the characteristic Venn diagram for propositions of that type. Recall our numbering of Venn diagrams for arguments:



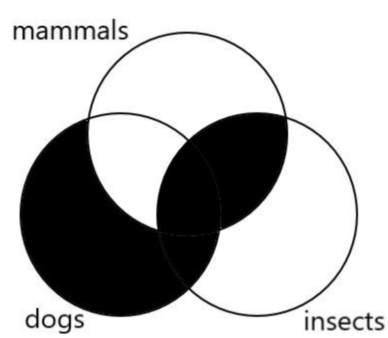
Here is what features the characteristic diagrams tell us we require for a Venn diagram to make the conclusion of a categorical syllogism true:

- A-Proposition: regions 2 and 5 shaded
- E-Proposition: regions 3 and 6 shaded
- I-Proposition: an x in either region 3 or region 6
- O-Proposition: an x in either region 2 or region 5

There will, of course, be shading or x's in other regions of the diagram. But for the purpose of interpreting the diagram that can all be ignored. All that matters is that the features required for the truth of the conclusion be present.
Consider, again, the following categorical syllogism:

> No mammals are insects
> <u>All dogs are mammals</u>
> No dogs are insects

The diagram for the premises we produced for it in the previous section was,



There are two questions we need to ask in order to determine whether the argument is valid:

1. What features does the diagram need to have to make the conclusion of the argument true?

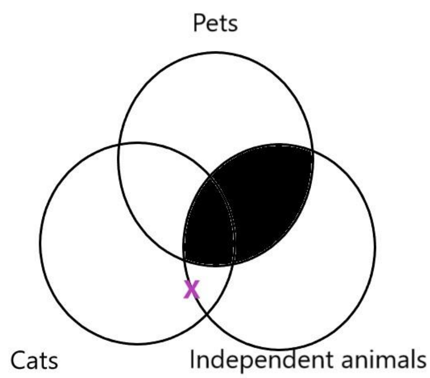
2. Are the required features present in the diagram?

Since the conclusion of the argument is an E-proposition, we know without even looking at the diagram that regions 3 and 6 need to be shaded in order for it to be true. And when we look at the diagram we can see that regions 3 and 6 are shaded. As a result, the diagram for the premises makes the conclusion true and, hence, the argument is valid.

Consider now the categorical syllogism

No pets are independent animals
<u>Some cats are not pets</u>
Some cats are independent animals

which we diagrammed as follows in the previous section:



Since the conclusion of the argument is an I-statement, we know we need an x in either region 3 or region 6 for it to be true. And there is no x in either region. There is, of course, an x on the line between regions 5 and 6. But that means there is something that is either in region 5 or region 6. But since there is no guarantee that this item is in region 6, the premises of the argument do not make the conclusion true. As a result, the argument is invalid.

Exercises Section 5.2

Using Venn Diagrams, determine whether or not each of the following categorical syllogisms is a valid argument. Indicate which feature of the diagram you rely on in making your determination.

1. Some internet startups are not big events

All new online courses are internet startups
Some new online courses are not big events

2. No people who live in glass houses are friends of mine
Some friends of mine are people who throw stones
No people who throw stones are people who live in glass houses

3. No late lunches are alcohol-free events
All late lunches are nap inducers
No nap inducers are alcohol-free events

4. No poorly trained pets are animals welcome in my boudoir
Some poorly trained pets are dangerous menaces
Some dangerous menaces are not animals welcome in my boudoir

5. All popular movie actors are excellent role models
Some excellent role models are logrolling champions
Some logrolling champions are popular movie actors

6. All men of good fortune are people who cause empires to fall
Some men of poor beginnings are not men of good fortune
Some men of poor beginnings are not people who cause empires to fall

7. All logic professors are people who are light on their feet
Some lords of the dance are people who are light on their feet
No lords of the dance are logic professors

8. Some profligate spenders are sportscar enthusiasts
All profligate spenders are appreciators of fine wines
Some appreciators of fine wines are sportscar enthusiasts

9. All shoe-phones are secret agent paraphernalia
No secret agent paraphernalia are items hidden beneath my floorboards
No items hidden beneath my floorboards are shoe-phones

10. Some disposable consumer goods are things that are bad for the environment
All things that are bad for the environment are products of sweatshop labor
Some products of sweatshop labor are disposable consumer goods

11. No flying pink elephants are nonfictional entities
All nonfictional entities are potential sources of conflict
Some potential sources of conflict are flying pink elephants

12. Some hare-brained schemes are occupational hazards
All hare-brained schemes are things on my bucket list

Some things on my bucket list are occupational hazards.

13. All fancy smartphones are miniature computers
    All slide rules are fancy smartphones
    All slide rules are miniature computers

14. All spicy vindaloos are blazing curries
    All meals prepared in my kitchen are blazing curries
    All meals prepared in my kitchen are spicy vindaloos

15. No normal mornings are times for relaxed meditation
    All times of pre-school crisis are times for relaxed meditation
    No times of pre-school crisis are normal mornings

3. Reducing the Number of Terms

A. Counting Terms

Consider the following syllogism:

Some donkeys are not well-tempered beasts
No ill-tempered beasts are literate readers
Some illiterate readers are non-donkeys

The first thing to note is that this is not a standard-form categorical syllogism. The problem is that rather than having three terms, each of which occurs twice, this argument contains six distinct terms: “donkeys,” “well-tempered beasts,” “ill-tempered beasts,” “literate readers,” “illiterate readers,” and “non-donkeys.” But the second thing to note is that these terms are not all independent of one another. What we have are three pairs of terms such that the members of each pair are complements of one another: each member of the pair denotes the class of things that fall outside the class denoted by the other member of the pair.

donkeys	non-donkeys
well-tempered beasts	ill-tempered beasts
literate readers	illiterate readers

This kind of pairing of terms suggests that although the argument is not, as it stands, a categorical syllogism, it might be equivalent to one.

In order to assess whether or not a syllogism might be equivalent to a standard-form categorical syllogism, the first thing one needs to do is to abbreviate the terms using upper case letters and the prefix “non.“ For example, we might use the following abbreviation schema:

D = donkeys

W = well-tempered beasts

L = literate readers.

This abbreviation schema also enables us to abbreviate the remaining terms in the argument as follows:

Non-D = non-donkeys

Non-W = ill-tempered beasts

Non-L = illiterate readers

The upshot is the following abbreviation of the original argument:

Some D are not W
No non-W are L
Some non-L are non-D

There are two comments worth making at this point. First, the fact that one can abbreviate an argument in this way using only three upper case letters and the prefix “non” is good, but not decisive, evidence that the argument is equivalent to a categorical syllogism. It could turn out that, because of how the terms are distributed in it, the argument is not equivalent to a categorical syllogism. And second, the choice of which term to abbreviate using the prefix “non” can at times be arbitrary. In the above example, we could just as easily have abbreviated “ill-tempered beasts” with “I” and “well-tempered beasts” with “non-I.”

B. Reducing Terms

The method we are going to use to transform arguments with more than three terms into standard-form categorical syllogisms involves the strategic use of the operations we developed in chapter 4: conversion, obversion, and contraposition. Recall: *conversion* involves switching the subject and predicate terms of a categorical proposition; *obversion* involves changing the quality of the proposition and replacing the predicate term with its complement; and *contraposition* involves switching the subject and predicate terms and replacing both with their complements. And in order to reduce the number of terms in a syllogism by means of deploying these operations, one has to be cognizant of the equivalences tabulated below:

Proposition	Obverse	Converse	Contrapositive
A: All S are P	Equivalent	Not equivalent	Equivalent

E: No S are P	Equivalent	Equivalent	Not equivalent
I: Some S are P	Equivalent	Equivalent	Not equivalent
O: Some S are not P	Equivalent	Not equivalent	Equivalent

.

Consider again the abbreviated argument we produced in the previous section:

P1: Some D are not W
P2: No non-W are L
C: Some non-L are non-D

Since the first premise, P1, is an O-statement, our table tells us it is equivalent to its contrapositive. To find the contrapositive of P1, we switch the subject and predicate terms and replace each with its complement. This yields the following:

P1*: Some non-W are not non-D

And since every categorical proposition is equivalent to its obverse, P2, an E-statement, is equivalent to its obverse. As above, to find the obverse of P2, we change the quality and replace the predicate with its complement, which yields the following:

P2*: All non-W are non-L.

We can represent the changes we have made as follows:

P1: Some D are not W → *contraposition* → P1*: Some non-W are not non-D
P2: No non-W are L → *obversion* → P2*: All non-W are non-L
C: Some non-L are non-D → *no change* → C: Some non-L are non-D

The argument that has resulted from this pair of operations contains three terms rather than the six we started with: "non-W," "non-D," and "non-L." Moreover, since each term occurs twice – and in different statements – and the major premise – in which the predicate of the conclusion occurs – is listed first, the result is a standard-form categorical syllogism that is equivalent to the original target argument. There are three comments worth making at this point:

1. There is usually more than one way to produce a standard-form categorical syllogism that is equivalent to the original. In the case at hand, we could have replaced both premises and the conclusion with their obverses:

   P1: Some D are not W → *obversion* → P1^: Some D are non-W
   P2: No non-W are L → *obversion* → P2^: All non-W are non-L
   C: Some non-L are non-D → *obversion* → C^: Some non-L are not D

Note that this again results in three terms, but a different set of terms than our first attempt yielded: “D,” “non-W,” and “non-L.”

2. As should be clear, there is no requirement that the three terms you end up with all lack the prefix “non.” All that matters is that you end up with exactly three. Any one of the following combinations would suffice:

    D, W, L
    D, W, non-L
    D, non-W, L
    D, non-W, non-L
    non-D, W, L
    non-D, W, non-L
    non-D, non-W, L
    non-D, non-W, non-L

3. You need to make sure that the operations you use yield statements that are equivalent to the statements you use them on. You might, for example, be tempted to use contraposition on C which would result in the following:

    C*: Some D are L

    But this would be a mistake, because C is an I-statement and the table of equivalences tells us that I-statements are not equivalent to their contrapositives.

Exercises Section 5.3

Abbreviate each argument using the indicated abbreviation schema. Using conversion, obversion, and contraposition transform the abbreviated argument into a logically equivalent categorical syllogism containing exactly three terms.

1. Some orangutangs are not big talkers
   <u>All primates without class are orangutangs</u>
   Some primates with class are not big talkers

   [O: orangutangs/ P: primates with class/ B: big talkers]

2. No insensitive people are grief counsellors
   <u>Some grief counsellors are extreme sports enthusiasts</u>
   No extreme sports enthusiasts are sensitive people

   [S: sensitive people/ G: grief counsellors/ E: extreme sports enthusiasts]

3. No sober people are binge eaters
<u>All drunk people are hesitant walkers</u>
No hesitant walkers are binge eaters

[S: sober people/ B: binge eaters/ H: hesitant walkers]

4. No wolverines are domesticated animals
<u>Some wolverines are noble beasts</u>
Some noble beasts are not feral animals

[W: wolverines/ D: domesticated animals/ N: noble beasts]

5. All places without borders are incorporated communities
<u>Some incorporated communities are villages</u>
Some villages are places with borders

[B: places with borders/ I: incorporated communities/ V: villages]

6. All people without morals are potential criminals
<u>Some politicians are not people with morals</u>
Some politicians are not people who are not potential criminals

[M: people with morals/ C: potential criminals/ P: Politicians]

7. All squirrels are low flyers
<u>Some undomesticated animals are high-flyers</u>
No domesticated animals are squirrels

[S: squirrels/ H: high-flyers/ D: domesticated animals]

8. Some people with limits are accountants
<u>All people without limits are daredevils</u>
Some people who are not daredevils are accountants

[L: people with limits/ A: accountants/ D: daredevils]

9. All pants other than jeans are bedazzled leg coverings
<u>No bedazzled leg coverings are clothes without flair</u>
No clothes with flair are jeans

[J: jeans/ B: bedazzled leg coverings/ F: clothes with flair]

10. Some election volunteers are voters
<u>All voters are domestic citizens</u>
Some foreigners are people who don’t volunteer during elections

[D: domestic citizens/ V: voters/ E: election volunteers]

11. No non-cats are non-humans
<u>All human beings are furry animals</u>
Some furless animals are cats

[C: cats/ H: human beings/ F: furry animals]

12. Some climate change accepters are carnivores
<u>All climate change deniers are pro-vaxxers</u>
Some anti-vaxxers are vegetarians

[V: vegetarians/ C: climate change accepters/ P: pro-vaxxer]

13. All unmarried people are conformists
<u>All people without ambition are married people</u>
All ambitious people are non-conformists

[M: married people/ C: conformists/ A: ambitious people]

14. All people without fear are shameless people
<u>All non-professionals are ashamed people</u>
All professionals are fearful people

[F: fearful people/ A: ashamed people/ P: professionals]

15. No unlocked doors are receptacles of ignorance
<u>All old minds are receptacles of wisdom</u>
No youthful minds are locked doors

[Y: youthful minds/ L: locked doors/ W: receptacles of wisdom]

Chapter 6: Causal Reasoning

1. Necessary and Sufficient Causation

Causation is a fundamental and universal feature of the world and our experience of it. At the macro-level at least, everything that happens is caused by what happened previously. And knowledge of causal regularities enables us to navigate the world and achieve our goals. Even successfully crossing the street requires knowing that the presence of a stop-sign or a red traffic light will cause most cars to stop, at least temporarily. **Causation** is a relation between events or facts that holds when an earlier fact or event makes a later fact or event happen. Consider, for example, a case in which the cue ball knocks the four-ball into the side pocket during a game of pool. We can distinguish between two separate events in this case: that of the cue ball striking the four-ball and that of the four-ball rolling into the side pocket. That these two events occur in succession is no mere coincidence: the earlier event – in which the cue ball strikes the four-ball – causes the later event – in which the four-ball roles into the side pocket – to happen. After all, other things being equal, if the cue ball had not struck the four-ball, the four-ball would not have rolled into the side pocket. Note: we refer to the earlier fact or event, which makes a subsequent fact or event occur, as the "**cause**" and to the later fact or event, which is caused by the earlier one, as the "**effect**." So, in our example, the event of the cue ball hitting the four-ball would be the cause and the event of the four-ball rolling into the side pocket would be the effect.

There are a number of different senses in which one fact or event might be said to cause another. Two senses of interest here include being a necessary cause and being a sufficient cause. In order to illuminate this distinction, it will prove fruitful to discuss the distinction between necessary and sufficient conditions more generally. To say that a condition A is **necessary** for condition B is to say that A is required for B, that B cannot be present or occur without the presence or occurrence of A. For example, being a mammal is a necessary condition for being a dog: after all, if a particular animal is not a mammal, then it cannot be a dog. To say that condition A is **sufficient** for condition B is to say that A is good enough for B, that the presence or occurrence of A ensures the presence or occurrence of B. For example, being a surgeon is sufficient for being a doctor: after all, anyone who is a surgeon must already be a doctor.

To say that a fact or event is a **necessary cause** of some effect is to say the fact or event is required to bring about the effect, that whenever the effect occurs, the fact or event must previously have occurred. Suppose, for example, that a fire occurs after the ignition of a fuel source in the presence of oxygen. The presence of oxygen counts as a necessary cause of the fire in this sense. After all, if one attempts to ignite a fuel source in the absence of oxygen, no fire will ensue. Moreover, if one wants to establish that some (type of) fact or event is ***not*** a necessary cause of some (type of) effect, all one needs to do is identify a case in which the effect occurs but the fact or event does not. For example, the fact that disorientation can occur without the prior consumption of alcohol shows that the consumption of alcohol is not a necessary cause of disorientation.

To say that a fact or event is a **sufficient cause** of some effect is to say that the occurrence of the fact or event is by itself good enough to bring about the effect, that whenever the fact or event occurs, the effect must subsequently occur. Striking a match in the presence of fuel and oxygen, for example, is a sufficient cause of the fire ensues. After all, whenever one lights match in the presence of fuel and oxygen, fire is the result. Moreover, if one wants to establish that some (type of) fact event is ***not*** a sufficient cause of some (type of) effect, all one needs to do is identify a case in which the fact or event occurs but the effect does not. For example, the fact that one can consume alcohol without subsequently becoming disoriented shows that the consumption of alcohol is not a sufficient cause of disorientation.

It is worth emphasizing that it does not follow from the fact that a fact or event is a necessary cause of an effect that it is also a sufficient cause of that effect. For example, although the presence of oxygen is a necessary cause of a fire, it is not a sufficient cause: by itself the presence of oxygen does not result in fire. There are lots of occasions in which oxygen is present but fires do not occur (which is a very good thing, from our perspective at least.) Moreover, it does not follow from the fact that a fact or event is a sufficient cause of an effect that it is also a necessary cause of that effect. For example, although striking a match in the presence of fuel and oxygen may be a sufficient cause of a fire, it is not a necessary cause. After all, a fire can be produced by flicking a lighter or focusing a beam of light through a magnifying glass – or even a lightning strike – in the very same circumstances. In some cases, however, we can identify a fact or event that is a **necessary and sufficient** cause of an effect, that is, a fact or event that is both is required for the occurrence of the effect and by itself good enough to produce the effect. Typically, only complex and general facts or events count as necessary and sufficient causes. So, while the presence of oxygen is may be too simple to be a sufficient cause of a fire and the striking of a match may be too specific to be a necessary cause of the fire, the general and complex fact that ignition occurred in the presence of fuel and oxygen may nevertheless count as a necessary and sufficient cause of the fire.

2. Mill's Methods

Causal reasoning includes, among other things, presenting reasons or evidence designed to identify the causes of various phenomena. Now it is true that by identifying the cause of a phenomenon, you thereby explain the phenomenon rather than providing an argument for it. But this is compatible with presenting an argument whose conclusion is that a certain fact or event is the cause of the phenomenon. For example, although the dry climate in Saskatoon may explain why acoustic guitars tend to develop cracks in this locale, we may still need to provide reasons or evidence that it is the dry climate which causes the cracks as opposed to some other fact or event. After all, the proposition that guitars tend to develop cracks in Saskatoon could be taken for granted while the proposition that the dry climate in Saskatoon is what causes this tendency might remain controversial. What is important to note is that the causal arguments at issue here are inductive arguments: the reasons and evidence offered are intended only to make the causal claims they support likely or probable rather than to render the truth of such claims certain.

Mill's methods – named after the 19th century philosopher John Stuart Mill – are techniques designed to identify the causes of various phenomena. Although these methods have been both subjected to trenchant criticism and substantially revised in various ways since their original development, they remain a useful introduction to causal reasoning. Mill himself introduced five distinct methods of identifying causes, of which we are going to focus on three here: the method of agreement, the method of difference, and the joint method of agreement and difference. We will consider each in turn.

A. Method of Agreement

The first of Mill's methods we are going to discuss is the **method of agreement**. This technique is designed to identify a fact or event that functions as a necessary cause of an effect under consideration. The method of agreement is deployed in cases in which there are multiple occurrences of some phenomenon. It consists of a systematic effort to identify a single factor that is common to each of these cases. And if some such fact or event can be identified, we can conclude that it is a necessary cause of the phenomenon at issue.

In order to see how the method of agreement works, it may prove useful to focus on a specific example. Suppose that Mary, Dora, and Rashid all came down with the flu and we were wondering what caused this to occur. We would proceed by first identifying candidate causal factors, and second determining which of these factors were present in each of the individual cases. Mary, let's suppose, went out in the rain and got a chill a few nights before coming with the flu, whereas Dora and Rashid stayed in that night and thereby avoided getting a chill. Dora and Rashid, refrained from eating their vegetables in the days prior to getting sick whereas Mary ate all of her vegetables. Finally, let's suppose that all of them visited their sick friend, Diego, earlier in the week and were thereby exposed to the flu virus. In light of these considerations, we might suppose we have three candidate causes of the phenomenon at issue: getting a chill, not eating vegetables, and being exposed to the flu virus. And we can indicate the presence or absence of these conditions in various cases in the following table, using "Y" to indicate a factor is present in a given case and "N" to indicate its absence:

Occurrence	Got a Chill	Didn't Eat Vegetables	Exposed to virus	Phenomenon (Flu)
Mary	Y	N	Y	Y
Dora	N	Y	Y	Y

Rashid	N	Y	Y	Y

Now it is important to recall that the method of agreement is designed to identify a necessary cause of a phenomenon, that is, a fact or event that is required to bring about the effect. As a result, any factor that is not present in some cases in which the effect in question occurs can be ruled out as a necessary cause of said effect. So, for example, because Mary got the flu despite eating her vegetables, not eating one's vegetables can be ruled out as a necessary cause of the flu; and because Dora and Rashid got the flu despite not getting a chill, getting a chill can be ruled out as a necessary cause as well. But because everyone who got the flu was previously exposed to the flu virus, the method of agreement tells us that being exposed to the flu virus is a necessary cause of getting the flu. Note: although an individual row in the table can establish that one of the factors is *not* a necessary cause of the phenomenon, only the table as a whole can establish that a factor *is* a necessary cause. Finally, it is worth emphasizing that the conclusion yielded by the method of agreement – that a certain fact or event is a necessary cause of some effect – is only probable given the evidence. There are a couple of reasons for this. First, it is possible that certain candidate causes of the phenomenon at issue were overlooked. And second, if a larger number of cases were considered, it could turn out that the factor identified as the necessary cause in the original sample of cases failed to be present in one or more members of the broader sample of cases, subsequently considered, in which the effect occurred.

B. Method of Difference

Next let us turn to Mill's **method of difference**. This technique is designed to identify a fact or event that functions as a sufficient cause of an effect under consideration. The method of difference is deployed in circumstances in which there is one case in which an effect is present and one case in which it is absent. It consists of a systematic effort to identify a single factor that is present in the case in which the effect occurs and absent in the case in which the effect does not occur. And if some such fact or event can be identified, then we can conclude that it is a sufficient cause of the phenomenon at issue.

In order to see how the method of difference works, it may prove useful to focus on a specific example. Suppose that Diego comes down with food poisoning but Dora does not and we were wondering what made Diego ill. We would proceed by identifying candidate causal factors and determining which of these factors was present in Diego's case but absent in Dora's case. Diego, let's suppose, gorged on the lasagna that had been left out on the counter overnight and followed that up with a healthy portion of a leftover raw pork dish. Dora joined Diego in feasting on the lasagna but was too full to have any of the pork. A few hours later, both of them made sandwiches with the peanut butter that had been in the fridge since they moved into their apartment. In light of these considerations, we might suppose we have three candidate causes of Diego's illness: eating the lasagna that was left out overnight, eating the raw pork dish, and making a sandwich with the mystery peanut butter. And we can indicate the presence or absence of these conditions in the following table:

Occurrence	**Left out lasagna**	**Peanut butter**	**Leftover raw pork**	**Phenomenon**
Dora	Y	Y	N	N
Diego	Y	Y	Y	Y

Recall: the method of difference is designed to identify a sufficient cause of a phenomenon, that is, a fact or event that is good enough by itself to bring about the effect. As a result, any factor that is present in cases in which the effect in question does not occur can be ruled out as a sufficient cause of that effect. So, for example, since Dora did not get food poisoning despite consuming both the lasagna that was left out overnight and the mystery peanut butter, both factors can be ruled out as sufficient causes of food poisoning. But since Diego ate the leftover raw pork dish and got sick, while Dora refrained from eating this dish and didn't get sick, the method of difference tells us that eating the leftover raw pork dish was a sufficient cause of food poisoning. Note: although an individual row in the table can establish that one of the factors is *not* a sufficient cause of the phenomenon, only the table as a whole can establish that a factor *is* a sufficient cause. Finally, the conclusion yielded by the method of difference – that a certain fact or event is a sufficient cause of some effect – is only probable given the evidence for the same reasons as noted above.

C. Joint Method of Agreement and Difference

Finally, we will consider Mill's **joint method of agreement and difference.** This technique is designed to identify a fact or event that functions as a necessary and sufficient cause of an effect under consideration. The joint method is deployed in circumstances in which there are multiple cases in which an effect is present and multiple cases in which it is absent. It consists of a systematic effort to identify a single factor that is present in all cases in which the effect occurs and absent in all cases in which the effect does not occur. And if some such fact or event can be identified, we can conclude that it is a necessary and sufficient cause of the phenomenon at issue.

As with the other methods, it will prove useful to focus on a specific example to see how the joint method of agreement and difference works. Let's suppose that one of Dora and Diego's other roommates, Jane, comes down with food poisoning too, but their fourth roommate, Rashid, does not. And we might wonder what made both Jane and Diego ill. We would proceed by identifying candidate causal factors and determining which of these factors was present in both Jane's and Diego's cases but absent in both Dora's and Rashid's cases. Let's assume that Dora and Diego's culinary decisions remain as characterized above. But let's suppose, in addition, that Jane ate both the suspect lasagna and the leftover raw pork dish, while Rashid sampled neither of them. And, moreover, let's suppose that Rashid made a sandwich with the mystery peanut butter while Jane refrained. We have the same three candidate causes of both Jane's and Diego's illnesses: eating the lasagna that was left out overnight, eating the raw pork dish, and making a sandwich using the peanut butter with mysterious origins. And we can indicate the presence or absence of these conditions in the following table:

Occurrence	Leftover lasagna	Peanut butter	Leftover raw pork	Phenomenon
Dora	Y	Y	N	N
Diego	Y	Y	Y	Y
Jane	Y	N	Y	Y
Rashid	N	Y	N	N

As noted above, the joint method of agreement and difference is designed to identify a necessary and sufficient cause of a phenomenon, that is, a fact or event that is both required to bring about the effect and good enough by itself to do so. As a result, any factor that is not present in some cases in which the effect in question occurs can be ruled out as a necessary cause of said effect and, hence, as a necessary and sufficient cause as well. So, since Jane got food poisoning without eating the mystery peanut butter, consuming said peanut butter can be ruled out as a necessary and sufficient cause of food poisoning. In addition, any factor that is present in cases in which the effect in question does not occur can be ruled out as a sufficient cause of that effect and, hence, as a necessary and sufficient cause as well. So, since Dora ate the lasagna without getting sick, and since both Dora and Rashid ate the peanut butter without getting sick, both factors can be ruled out as sufficient causes of food poisoning. But since both Jane and Diego are the leftover raw pork dish and got sick, while Dora and Rashid refrained from eating this dish and didn't get sick, the joint method tells us that eating the leftover raw pork dish was a necessary and sufficient cause of food poisoning. Note: although an individual row in the table can establish that one of the factors is *not* a necessary and sufficient cause of the phenomenon, only the table as a whole can establish that a factor *is* a necessary and sufficient cause.

Exercises Section 6.2

For each of the following questions (i) identify the cause suggested by information presented in the table; (ii) identify what sense of causation is at issue; and (iii) identify which of Mill's methods is being utilized.

1.

	Possible Conditions				
Occurrence	*A*	*B*	*C*	*D*	**Phenomenon**
1	Y	N	Y	Y	Y

2	N	Y	Y	Y	Y
3	N	Y	Y	N	Y
4	Y	Y	Y	Y	Y

2.

	Possible Conditions				
Occurrence	*A*	*B*	*C*	*D*	**Phenomenon**
1	Y	Y	N	Y	Y
2	N	Y	N	Y	N
3	N	Y	N	N	N
4	Y	Y	N	N	Y
5	Y	Y	N	Y	Y
6	N	Y	N	N	N

3.

	Possible Conditions				
Occurrence	*A*	*B*	*C*	*D*	**Phenomenon**
1	Y	Y	N	Y	N
2	Y	Y	Y	Y	Y

4.

	Possible Conditions				
Occurrence	*A*	*B*	*C*	*D*	**Phenomenon**
1	N	Y	Y	N	N
2	Y	Y	Y	N	N
3	N	N	Y	Y	Y
4	Y	Y	Y	N	N
5	N	N	Y	Y	Y
6	Y	N	Y	Y	Y

5.

	Possible Conditions						
Occurrence	*A*	*B*	*C*	*D*	*E*	*F*	**Phenomenon**
1	Y	Y	Y	Y	N	Y	N
2	Y	Y	Y	Y	Y	Y	Y

6.

	Possible Conditions				
Occurrence	*A*	*B*	*C*	*D*	**Phenomenon**
1	N	Y	Y	N	Y
2	N	Y	N	Y	Y
3	N	Y	N	Y	Y
4	N	Y	Y	N	Y

7.

	Possible Conditions						
Occurrence	*A*	*B*	*C*	*D*	*E*	*F*	**Phenomenon**
1	N	Y	N	N	Y	Y	Y
2	Y	Y	N	N	Y	Y	Y
3	N	N	N	Y	Y	Y	Y
4	Y	N	N	Y	Y	Y	Y
5	N	N	N	Y	Y	Y	Y
6	Y	N	N	N	Y	N	Y

8.

	Possible Conditions						
Occurrence	*A*	*B*	*C*	*D*	*E*	*F*	**Phenomenon**
1	N	N	N	N	Y	Y	N
2	N	Y	Y	N	Y	N	Y
3	N	Y	N	Y	Y	N	Y
4	N	Y	Y	Y	N	N	Y
5	Y	N	Y	Y	Y	Y	N
6	Y	N	N	N	N	Y	N
7	Y	Y	Y	Y	N	N	Y
8	Y	N	N	N	Y	Y	N

9.

	Possible Conditions						
Occurrence	*A*	*B*	*C*	*D*	*E*	*F*	**Phenomenon**
1	Y	Y	Y	Y	Y	Y	Y
2	N	Y	Y	Y	Y	Y	N

10.

	Possible Conditions						
Occurrence	*A*	*B*	*C*	*D*	*E*	*F*	**Phenomenon**
1	Y	N	N	Y	N	N	Y
2	Y	N	N	Y	Y	Y	Y
3	N	Y	N	Y	N	Y	N
4	Y	N	Y	N	Y	N	Y
5	N	Y	Y	Y	N	N	N
6	Y	N	N	N	N	N	Y
7	N	Y	N	Y	Y	Y	N
8	N	Y	N	N	Y	Y	N

(i) Complete each of the following tables using the information provided in the word problems, (ii) interpret each table to determine which condition is the cause of the phenomenon at issue, and (iii) identify in what sense the condition in question is a cause.

11. Mary, Fred, Jada, and Rashid get trapped outside the post-apocalyptic safe-zone. Mary drinks zombie-infected water, breathes zombie-infected air, and has some zombie-infected cuts, but does not eat zombie-infected food. Fred has zombie-infected cuts and breathes zombie-infected air, but doesn't eat or drink anything zombie-infected. Jada and Rashid eat zombie-infected food. Jada but not Rashid has zombie-infect cuts, and Rashid but not Jada drinks zombie-infected water. They both, however, breathe zombie-infected air. They all come down with the zombie plague. What caused them to become zombies?

Occurrence	*Drink Infected Water*	*Eat Infected Food*	*Breathe Infected Air*	*Infected Cuts*	**Zombie Plague**
Mary					
Fred					
Jada					
Rashid					

12. A pair of twins, Mary and Jada, sleep in twin beds in the same room. Mary and Jada both drink a glass of wine before bed. Mary uses her tablet and does homework before bed, but although Jada does her homework before bed as well, she does not use her tablet. Finally, both Mary and Jada have a snack before bed. Mary suffers from insomnia but Jada does not. What condition causes Mary's insomnia?

Occurrence	*Wine*	*Tablet*	*Snack*	*Homework*	**Insomnia**
Mary					
Jada					

13. An investigation of an incident at the University of Saskatchewan space station reveals that Fred, Jada, and Diego have been lost in space while Mary, Rashid, and Dora remain safely inside the station. Mary, Fred, and Jada had defective transmitters in their spacesuits, while Rashid, Dora, and Diego used suits with properly functioning transmitters. Mary and Jada both exited the station via a defective airlock and both used defective space harnesses, but Mary was polite to the station AI HAL whereas Jada insulted HAL. Fred, Rashid, and Dora all exited the station using a well-functioning airlock and they all used defective harnesses. Diego insulted HAL, exited by a defective airlock, and used a properly

functioning harness. Fred insulted HAL, but Rashid and Dora were polite to HAL. What caused Fred, Jada, and Diego to be lost in space?

Occurrence	*Defective Transmitter*	*Defective Airlock*	*Defective Harness*	*Insulted HAL*	**Lost in Space**
Mary					
Fred					
Jada					
Rashid					
Dora					
Diego					

14. Mary, Fred, Jada, and Rashid take Alward's logic quiz. Mary comes to class regularly but never participates, studies hard, and gives Alward generous tips. Fred and Rashid never study but they come to class regularly. Jada studies hard but doesn't come to class regularly, although she participates when she does and always leaves a generous tip. Fred rarely participates but Rashid participates regularly, and they both leave generous tips. They all get grades of 90% or higher on their quiz. What caused them to get this grade?

Occurrence	*Generous Tip*	*Study Hard*	*Come to Class*	*Participate*	**90% grade**
Mary					
Fred					
Jada					
Rashid					

15. Both Mary and Jada have the same model and make of laptop from the same year. Neither of them ever updates their software, and they both share their passwords with anyone who asks. Mary often downloads online files on to her laptop but never spills anything on it. Jada also often downloads online files onto her laptop and regularly spills things on it. Jada's laptop constantly crashes but Mary's does not. What causes Jada's laptop to crash?

Occurrence	*No Updates*	*Spills*	*Downloads*	*Passwords*	**Crashes**
Mary's Laptop					
Jada's Laptop					

16. Mary, Fred, Jada, Rashid, Dora, and Diego were all trying to catch a flight to Hawaii, but while Mary, Rashid, and Diego all made the flight, Fred, Jada and Dora missed it. Mary and Rashid left early for the airport, and Mary and Diego brought only carry on luggage. Fred had valid ID and carry on luggage, but he

didn't leave early for the airport. Dora, Diego, and Rashid all had valid ID, and Dora and Rashid checked in online while Diego did not. Jada checked in online, had only carry on luggage and brought valid ID, but she did not leave early for the airport. Neither Mary nor Fred checked in online, and Mary didn't have valid ID either. Neither Rashid nor Dora brought only carry on luggage, and while Diego left early for the airport, Dora did not. What caused Mary, Rashid, and Diego to make their flight.

Occurrence	*Left Early for Airport*	*Checked in Online*	*Carry On Luggage*	*Valid ID*	**Make Flight**
Mary					
Fred					
Jada					
Rashid					
Dora					
Diego					

17. Mary, Fred, Jada, and Rashid only have leftover takeout containers of food in their fridges from the meals they have together in budget restaurants. Mary has some king pao chicken, half of a big mac, and some fried okra and black-eyed peas but no lasagna. Fred and Jada have lasagna but Rashid does not. Jada has half of a big mac, king pao chicken but no fried okra and black-eyed peas. Fred has fired okra and black-eyed peas and half a big mac, but no king pao chicken. Rashid has king pao chicken, half of a big mac, but not fried okra and black-eyed peas. They all have unpleasant odors in their fridges. What caused this odor?

Occurrence	*King Pao Chicken*	*Big Mac*	*Lasagna*	*Fried Okra and Black-Eyed Peas*	**Unpleasant Fridge Odor**
Mary					
Fred					
Jada					
Rashid					

18. Both Mary and Jada get a puppy from the same litter. Mary enrolls her puppy in obedience classes and sends him to Doggy daycare. Jada sends her puppy to doggy daycare, crate trains him, and enrolls him in obedience classes. They both socialize their puppies with other people, and take them on regular walks. Mary allows her puppy to sleep in her bed rather than crate training him and Mary and Jada both give their puppies plenty of chew toys. Mary's puppy barks at strangers but Jada's puppy does not. What causes Jada's puppy to be good with strangers?

Occurrence	*Obedience Classes*	*Crate Training*	*Doggy Daycare*	*Human Socialization*	*Regular Walks*	*Chew Toys*	**Barks at Strangers**
Mary's Puppy							
Jada's Puppy							

19. Mary, Fred, Jada, Rashid, Dora, and Diego all try out for the Slytherin House quidditch team, but while Mary, Fred, and Rashid make the team, Jada, Dora, and Diego do not. Mary and Jada both tried to bribe the team captain, but while Mary gave the captain a love potion, Jada did not. Fred, Dora, and Diego attended all of the practice sessions, but while Fred and Diego neglected their schoolwork, Dora did not. Rashid attended all the practices, neglected his schoolwork, tried to bribe the team captain, and gave the captain a love potion. Neither Dora nor Diego tried to bribe the team captain or gave him a love potion. Mary and Jada both attended practices regularly and neither of them neglected their schoolwork. Fred did not try to bribe the team captain, but did give the captain a love potion. What caused Mary, Fred, and Rashid to make the team?

Occurrence	*Attend Practices*	*Neglect Schoolwork*	*Bribe Team Captain*	*Love Potion*	**Make Quidditch Team**
Mary					
Fred					
Jada					
Rashid					
Dora					
Diego					

20. Mary, Fred, Jada, Rashid, Dora, and Diego all go to an amusement park. Mary paid her admission, spent $20 on food, did not make a wish or get trapped on a ride, did a good deed, but did not set a high score. Fred, Jada, and Rashid each spent $20 on food but neither Dora nor Diego spent that much. Dora, Diego, and Rashid set high scores and paid their admission. Fred, Jada, Rashid, Dora, and Diego all got trapped on rides but none of them did good deeds. Fred paid his admission, made a wish, but did not set a high score. Jada and Rashid made wishes but Dora and Diego did not. And Jada set a high score and paid her admission. As they leave, they each receive a coupon for free admission next time they come to the park. What caused them to receive this coupon?

Occurrence	*Paid Admission*	*Spent $20 on Food*	*Made a Wish*	*Trapped on Ride*	*Did Good Deed*	*Set High Score*	**Free Admission Coupon**
Mary							
Fred							

Jada							
Rashid							
Dora							
Diego							

21. Mary and Jada both try to rob identical branches of the same bank. Mary and Jada both case the joint and wear a mask and gloves, and Jada subdues the guards. Mary listens to a police scanner and knows the vault combination. Jada also knows the vault combination but ignores the police scanner. Mary subdues the guards and they both have a getaway car waiting. Mary's successfully escapes a large sum of money whereas Jada's robbery is unsuccessful. What causes Mary's heist to be successful?

Occurrence	*Cases the Joint*	*Mask and Gloves*	*Subdues Guards*	*Police Scanner*	*Vault Combination*	*Getaway Car*	**Successful robbery**
Mary's Heist							
Jada's Heist							

22. Mary, Fred, Jada, Rashid, Dora, Diego, Namid, and Mikom all go to the skateboard park. Fred, Diego, Rashid and Mikom all get injured but, Mary, Jada, Dora, and Namid do not. Mary, Fred, and Jada wear safety gear and stay hydrated, but while Fred and Jada practice at home, Mary does not. Rashid doesn't pay attention and doesn't avoid the half-pipe, but does stick to easy tricks. Dora and Namid avoid the half-pipe and stick to easy tricks, but while Dora wears safety gear and pays attention, Namid does neither. Mary and Fred pay attention while Jada does not, Mary and Jada avoid the half-pipe while Fred does not, and Jada and Fred stick to easy tricks while Mary does not. Mikom wears safety gear and pays attention, but doesn't avoid the half-pipe and doesn't stick to easy tricks. Diego and Rashid both wear safety gear and practice at home, but neither of them stays hydrated. Namid and Mikon both stay hydrated but neither of them practices at home. Diego doesn't pay attention, doesn't avoid the half-pipe, and doesn't stick to easy tricks. Dora stays hydrated but doesn't practice at home. What caused Mary, Jada, Dora, and Namid to avoid injury?

Occurrence	*Safety Gear*	*Pay Attention*	*Stay Hydrated*	*Practice at Home*	*Avoid Half-pipe*	*Easy Tricks*	**Avoid Injury**
Mary							
Fred							
Jada							
Rashid							
Dora							
Diego							

Namid							
Mikom							

23. Mary, Fred, Jada, Rashid, Dora, and Diego all go to a dark magic convention. Fred touches a cursed monkey paw, rubs a magic lamp and drinks a magic potion, but doesn't go to a séance or have a spell cast on him or receive the evil eye. Mary, Jada, and Rashid go to a séance but Dora and Diego do not. Dora touches a cursed monkey paw, does not rub a magic lamp, receives the evil eye, has a spell cast on her, and drinks a magic potion. Mary, Jada, and Rashid receive the evil eye and Diego does not, but they all have a spell cast on them. Mary and Jada touch the cursed monkey paw and drink a magic potion, but Jada rubs a magic lamp while Mary does not. Rashid touches a cursed monkey paw and drink a magic potion, but does not rub a magic lamp. And Diego rubs a magic lamp and drinks a magic potion, but does not touch a cursed monkey paw. At the end of the night they are all transformed into chickens. What caused them to be so transformed?

Occurrence	*Monkey Paw*	*Magic Lamp*	*Evil Eye*	*Spell Cast*	*Drink Potion*	*Séance*	**Turned into Chicken**
Mary							
Fred							
Jada							
Rashid							
Dora							
Diego							

Chapter 7: Propositional Logic

1. Symbols and Translations

In this section, we will learn to translate sentences of English into the symbolic notation of propositional logic. The point of doing this is to make the logically relevant features – those features which contribute to validity and invalidity – of propositions and arguments clear and prominent, and to move those features that are logically irrelevant to the background. In categorical logic, the fundamental elements were terms: expressions which denote classes of things. In propositional logic, in contrast, the fundamental element is the statement or proposition. As before, we will be using the terms "statement" and "proposition" interchangeably in this chapter.

A fundamental distinction in propositional logic is the distinction between simple statements and compound statements. A **simple statement** is a statement that doesn't contain any other statement as a part or component. A **compound statement**, in contrast, is a statement that does contain another statement as a component. Consider the following pair of statements:

Logic is exciting   If logic is exciting then I'll eat my hat

The first member of the pair – "Logic is exciting" – is a simple statement: it does not contain another statement as a part. The second member of the pair – "If logic is exciting then I'll eat my hat" – is a compound statement: it contains two (other) statements – "logic is exciting" and "I'll eat my hat" – as components.
In propositional logic, we translate simple statements using uppercase letters of the alphabet. The following table provides some examples:

ENGLISH STATEMENT	TRANSLATION
Logic is exciting	L
Snow is white	S
Alward is wearing brown shoes	B

It is worth emphasizing that it does not matter what uppercase letter you use to translate a simple statement. We could just as easily have translated "Logic is exciting" using "E". But it is easiest to choose a letter that helps remind you of the English statement you have translated. However, if you are translating a group of statements in which a given simple statement occurs more than once, you need to translate it using the same letter each time.

In propositional logic, compound statements are translated using operators. **Operators** are symbols used to connect simple statements in propositional logic. The operators of propositional logic are listed in the following table:

OPERATOR	NAME	STATEMENT TYPE
~	tilde	negations
&	ampersand	conjunctions
v	vee	disjunctions
->	arrow	conditionals
<->	double arrow	biconditionals

The **main operator,** in a propositional logic translation, is the operator (in a compound statement) that applies to or governs everything else in the statement. The main connective determines what type of statement we are dealing with.

A **negation** is a statement that has a tilde as its main operator. The tilde is used to translate English expressions such as “not,” “it is not the case that,” and “it is false that,” among others. The following table provides some examples:

ENGLISH STATEMENT	TRANSLATION
Logic is not exciting	~L
It is not the case that snow is white	~S
It is false that Alward is wearing brown shoes	~B

Please note that the tilde always goes in front – that is, to the left – of the statement it negates.

A **conjunction** is a statement that has the ampersand as its main operator. The ampersand is used to translate English expressions such as “and,” “also,” “but,” “although,” and “however,” among others. The following table provides some examples:

ENGLISH STATEMENT	TRANSLATION
Logic is exciting and snow is not white	L & ~S
Snow is not white, but Alward is wearing brown shoes	~S & B

It is worth noting that the ampersand (and the remaining operators) goes in between the pair of statements it connects, in contrast to the tilde which goes in front of the single statement it modifies. It is also worth noting that despite the differences in meaning in English between “and” and “but,” for example, they both get translated by the ampersand in propositional logic. The symbolic language of propositional logic is simply not rich enough to capture these differences. But even so, differences of this kind rarely make a difference to the validity of arguments and so most of the time can be safely ignored.

A **disjunction** is a statement that has a vee as its main operator. The vee is used to translate "or" and "unless." The following table provides some examples:

ENGLISH STATEMENT	TRANSLATION
Alward is wearing brown shoes unless Logic is not exciting	B v ~L
Either snow is not white or Alward is wearing brown shoes and logic is exciting	~S v (B & L)

There are two things worth noting here. First, "unless" is equivalent in meaning to "if not" and, as a result, students are often tempted to translate it using the arrow. But students who attempt to do so often get the translation wrong; it is always easier to translate "unless" with the vee. Second, parentheses are important punctuation marks in propositional logic. Consider the following pair of symbolic statement, differing only in the placement of the parentheses:

(A v B) & C   A v (B & C)

In the first statement, the main operator is the ampersand, which makes it a conjunction; in the second statement, the main operator is the vee, which makes it a disjunction.

A **conditional** is a statement that has an arrow as its main operator. The arrow is used to translate "if … then … ," "only if" and similar expressions that indicate a conditional statement. In a conditional statement, the statement that occurs before (or to the left of) the arrow is the **antecedent**; the statement which occurs after (or to the right of) the arrow is the **consequent**. In translating with the arrow, it is important to make sure you figure out which statement is the antecedent and which is the consequent. The following rule of thumb is helpful in this regard:

> A statement that follows an "if" by itself (not preceded by the word "only") is the antecedent of the conditional; a statement that follows "only if" is the consequent of the conditional.

The following table provides some examples of translations using the arrow:

ENGLISH STATEMENT	TRANSLATION
Alward is not wearing brown shoes if snow is white	S -> ~B
Logic is not exciting only if Alward does not wear brown shoes	~L -> ~B
If either snow is white or logic is exciting, then Alward doesn't wear brown shoes	(S v L) -> ~B

The arrow can also be used to translate statements including the expressions "necessary condition" and "sufficient condition." In translating such expressions, always make sure that the statement describing the sufficient condition is the antecedent of the conditional and that the statement describing the necessary condition is the consequent of the conditional. For example, the statement "Being a doctor is a necessary condition for being a surgeon" is correctly translated as "S -> D" rather than "D -> S".

A **biconditional** is a statement that has a double arrow as its main operator. The double arrow is used to translate the expressions "if and only if," "just in case," and "is a sufficient and necessary condition for." The following table provides some examples of translations using the double arrow:

ENGLISH STATEMENT	TRANSLATION
Alward is wearing brown shoes if and only if snow is not white	B <-> ~S
Logic's being exciting is a necessary and sufficient condition for Alward's wearing brown shoes	L <-> B

In translating conditionals and biconditionals, it is important to note the difference between "only if," on the one hand, and "if and only if," on the other. The former gets translated using the arrow whereas the latter gets translated using the double arrow.

Care needs to be taken when translating the negations of conjunctions and disjunctions, statements that contain expressions such as "not either," "not both," "either not," and "both not." The following table contains examples illustrating the correct way to translate statements containing such expressions:

ENGLISH STATEMENT	TRANSLATION
It's not the case that either snow is white or grass is green	~(G v S)
Either snow is not white or grass is not green	~G v ~S
It's not the case that both snow is white and grass is green	~(S & G)
Both snow is not white and grass is not green	~S & ~G

One might be tempted to think that this is much ado about nothing because "~(G v S)" is equivalent to "~G v ~S" and "~(S & G)" is equivalent to "~S & ~G." But this would be incorrect. In propositional logic, the equivalences that hold are the following:

1. "~(G v S)" is equivalent to "~S & ~G"

2. "~(S & G)" is equivalent to "~S v ~G"

After all, to deny that either of two statement is true is to claim that both of them are false. And to deny that both of two statements are true is to claim that either one or the other of them (or both) is false.

Exercises Section 7.1

Translate each of the following statements into the symbolic notation of propositional logic, using the indicated symbolization keys.

1. Broken bones are not funny. [B: broken bones are funny]

2. Although it's snowing, it's not cold. [S: it's snowing/ C: it's cold]

3. It's not cold or it's snowing [S: it's snowing/ C: it's cold]

4. John is cold and wet unless it's not raining. [C: John is cold/ W: John is wet/ R: it's raining]

5. John is cold and wet, however it's not raining. [C: John is cold/ W: John is wet/ R: it's raining]

6. John is neither cold nor wet. [C: John is cold/ W: John is wet]

7. It's not the case that both Poilievre is Trudeau's most formidable opponent and Singh is Trudeau's most formidable opponent. [P: Poilievre is Trudeau's most formidable opponent/ S: Singh is Trudeau's most formidable opponent]

8. Either Poilievre is not Trudeau's most formidable opponent or Singh is not Trudeau's most formidable opponent. [P: Poilievre is Trudeau's most formidable opponent/ S: Singh is Trudeau's most formidable opponent]

9. John is both not cold and not wet. [C: John is cold/ W: John is wet]

10. Although Trump is neither in the White House nor on twitter, he retains political power. [W: Trump is in the White House/ T: Trump is twitter/ P: Trump retains political power]

11. Although Trump is not in the White House, he is not on twitter unless he retains political power. [W: Trump is in the White House/ T: Trump is ontwitter/ P: Trump retains political power]

12. If a rolling stone gathers no moss, then we either have to gather moss some other way or do without it. [R: a rolling stone gathers no moss/ G: we have to gather moss in some other way/ W: we have to do without moss]

13. A rolling stone gathers no moss if we both have to gather moss some other way and do without it. [R: a rolling stone gathers no moss/ G: we have to gather moss in some other way/ W: we have to do without moss]

14. A rolling stone doesn't gather any moss only if we either have to gather moss some other way or do without it. [R: a rolling stone gathers no moss/ G: we have to gather moss in some other way/ W: we have to do without moss]

15. If the sky is falling, then Chicken Little goes outside only if s/he is wearing a helmet. [S: the sky is falling/ O: Chicken Little goes outside/ H: Chicken Little is wearing a helmet]

16. The sky is falling, if Chicken Little goes outside only if s/he is wearing a helmet. [S: the sky is falling/ O: Chicken Little goes outside/ H: Chicken Little is wearing a helmet]

17. If Ignatius accepts the invitation to my soiree only if Constantine is out of the country, then either Constantine is not out of the country or my soiree is not a fiasco. [I: Ignatius accepts the invitation to my soiree/ C: Constantine is out of the country/ F: my soiree is a fiasco]

18. Either Ignatius accepts the invitation to my soiree if Constantine is out of the country or both Constantine is not out of the country and my soiree is not a fiasco. [I: Ignatius accepts the invitation to my soiree/ C: Constantine is out of the country/ F: my soiree is a fiasco]

19. Ichabod gets the promotion if and only if he's neither incoherent nor malodorous. [I: Ichabod is incoherent/ M: Ichabod is malodorous/ P: Ichabod gets the promotion]

20. Ichabod's being both incoherent and malodorous is necessary and sufficient for his not getting the promotion. [I: Ichabod is incoherent/ M: Ichabod is malodorous/ P: Ichabod gets the promotion]

21. If Ichabod gets the promotion just in case he's not incoherent, then Ichabod is not malodorous. [I: Ichabod is incoherent/ M: Ichabod is malodorous/ P: Ichabod gets the promotion]

22. It's not the case that both Haim deserves a Grammy just in case Harry Styles doesn't and Billie Eilish deserves a Grammy only if Beyoncé doesn't. [H: Haim deserves a Grammy/ S: Harry Styles deserves a Grammy/ E: Billie Eilish deserves a Grammy/ B: Beyoncé deserves a Grammy]

23. Either Haim deserves a Grammy if Harry Styles does as well or neither Billie Eilish nor Beyoncé deserve a Grammy. [H: Haim deserves a Grammy/ S: Harry Styles deserves a Grammy/ E: Billie Eilish deserves a Grammy/ B: Beyoncé deserves a Grammy]

24. If either Vanessa doesn't take up kick boxing or Fred is not both overworked and underpaid, then Fred takes up yoga if and only if he's not underpaid. [V: Vanessa takes up kick boxing/ W: Fred is overworked/ P: Fred is underpaid/ Y: Fred takes up yoga]

25. Both Vanessa takes up kick boxing and either Fred takes up yoga or he's overworked, only if Fred's not being overworked is necessary and sufficient for his being underpaid. [V: Vanessa takes up kick boxing/ W: Fred is overworked/ P: Fred is underpaid/ Y: Fred takes up yoga]

26. If all I do I do for you, then it follows that I do something for you only if it's either the case that I do nothing or that you don't exist. [A: all I do I do for you/ S: I do something for you/ D: I do something/ E: you exist]

27. If I don't do something for you only if both all I do I do for you and I do nothing, then you exist just in case it's not true that all I do I do for you. [A: all I do I do for you/ S: I do something for you/ D: I do something/ E: you exist]

28. Although the cattle are neither dying nor thriving only if we're getting moderate amounts of rainfall, if feed production is lower than usual then the cattle are either starving or not undergoing typical weight gain. [D: the cattle are dying/ T: the cattle are thriving/ M: we're getting moderate amounts of rainfall/ F: feed production is lower than usual/ S: the cattle are starving/ W: the cattle are undergoing typical weight gain]

29. We're not getting moderate amounts of rainfall just in case not both the cattle are not thriving and feed production is lower than usual; however, that cattle are not undergoing typical weight gain is necessary and sufficient for their starving or dying. [D: the cattle are dying/ T: the cattle are thriving/ M: we're getting moderate amounts of rainfall/ F: feed production is lower than usual/ S: the cattle are starving/ W: the cattle are undergoing typical weight gain]

30. McGee exists just in case someone perceives him if and only if McGee's being perceived is necessary and sufficient for his not being a perceiving subject; but McGee is both a perceiving subject and is perceived by someone unless he neither exists nor does anyone else. [M: McGee exists/ P: someone perceives McGee/ S: McGee is a perceiving subject/ E: someone other than McGee exists]

2. Natural Deduction

In this section, we will be developing the method for evaluating the validity of arguments known as natural deduction. **Natural deduction** consists in using inference rules to derive the conclusion of an argument from its premises. If you can derive the conclusion of an argument from its premises in this way, you have successfully shown the argument to be valid. It is worth noting, however, that failure to derive the conclusion from its premises does not establish that an argument is invalid. After all, it might still be possible to derive the conclusion despite the inability of a particular person to figure out how to do so. The

inference rules used in natural deduction are valid argument forms: arrangements of statement variables and operators such that uniform substitution of statements in place of variables results in an argument. Consider, for example, the following formula:

p v (*q* -> *p*)

The symbols "*p*" and "*q*" are variables in this formula. Just as "x" and "y" can stand for any number in the mathematical formula,

2x + y,

the "*p*" and "*q*" can stand for any statement in the symbolic language of propositional logic in the formula,

p v (*q* -> *p*).

Suppose, for example, we substitute "~B" for "*p*" and "(A <-> C)" for "*q*" in this formula. The result would be the following statement:

~B v ((A <-> C) -> ~B).

As we shall see, the application of inference rules simply consists in substituting statements for variables in the way.

A. Basic Inference Rules

A proof or **derivation** is a numbered sequence of propositions, each of which is either a premise of the argument under consideration or is derived from preceding propositions by the application of an inference rule. The last line of a derivation is the conclusion of the argument. As result, what you can derive, and how you can derive it, depends on exactly what inference rules are available to you. In the logical system developed here, there will be 11 rules: an introduction and elimination rule for each operator, together with a very simple rule of convenience we will call "Reiteration." In this subsection, we will consider the more basic inference rules which do not involve **sub-derivations**, that is, derivations within derivations. In the next subsection, we will consider the more complex sub-derivation rules. *Note: the rules discussed in this section cannot be applied to statements which occur in* ***completed*** *sub-derivations.*

The basic inference rules we will consider are the following:

Reiteration (R)

p
p

Ampersand Elimination (&E)

p & q - OR - *p & q*
p *q*

Ampersand Introduction (&I)

p

Vee Introduction (vI)

p - OR - *p*

q
p & q

p v q *q v p*

Arrow Elimination (->E)
p -> q
p
q

Vee Elimination (vE)
(p -> r) & (q -> r)
p v q
r

Double-Arrow Introduction (<->I)
(p -> q) & (q -> p)
p <-> q

Double-Arrow Elimination (<->E)
p <-> q
(p -> q) & (q -> p)

These basic inference rules tell us that if we already have statements in our derivations of the forms above the line in the rule, then we can introduce a new statement in our derivation of the form below the line. We will consider each of these rules in turn.

According to **reiteration**,

Reiteration (R)
p
p

we can simply introduce again at a new line any statement that has already appeared previously in the derivation. Suppose we want to construct a derivation for the following argument:

A <-> B
C & D
A <-> B

We begin by numbering the premises and writing “P” (for “premise”) to indicate their status as such:

1. A <-> B P
2. C & D P

Our goal is to introduce the conclusion at a line in the derivation as a result of the application of various inference rules to statements already appearing in the derivation (including themselves derived by means of these rules). And the derivation is complete once we have done so. Since every statement of propositional logic is of the form

p,

the rule reiteration tells us that we can repeat any line that has already appeared in a derivation. As a result, we can complete our derivation as follows:

1. A <-> B	P
2. C & D	P
3. A <-> B	R: 1

Every time we introduce a new line in a derivation we have to justify it. This involves indicating the rule of inference we used and the earlier line(s) in the derivation we applied it to. In the case at hand, we applied the rule *reiteration (R)* to line 1 of the derivation.

According to **ampersand elimination** (&E),

Ampersand Elimination (&E)

p & q - *OR* - *p & q*
p *q*

if we have a conjunction at a line in our derivation, we can introduce either conjunct of the conjunction at a new line in the derivation. Suppose we wanted to complete a derivation for the following argument:

A v (B & C)
(A v B) & C
C

We would begin our derivation as before:

1. A v (B & C)	P
2. (A v B) & C	P

Now in order to use ampersand elimination, we must already have a conjunction at a line in our derivation, that is, a statement of the form "*p* & *q*". Line 2 has such a statement in which "(A v B)" is substituted for "*p*" and "C" is substituted for "*q*". (Note: the statement at line 1 has the form "*p* v *q*" rather than "*p* & *q*" and so ampersand elimination cannot be applied to it). As a result, we can continue our derivation as follows:

1. A v (B & C)	P
2. (A v B) & C	P
3. C	&E: 2

And since the statement at line 3 is the conclusion of the argument at issue, the derivation is now complete.

According to **ampersand introduction** (&I),

Ampersand Introduction (&I)

p
q

p & q

if we have a statement of any kind at a line in our derivation and a statement of any kind at another line in our derivation, we can introduce the conjunction of those two statements at a new line in the derivation. Suppose we wanted to complete a derivation for the following argument:

(C <-> D) & F
<u>G & (H v I)</u>
F & G

We would begin the derivation in the usual way:

1. (C <-> D) & F	P
2. G & (H v I)	P

The first thing to note is that the two conjuncts of the conjunction we wish to derive already appear as conjuncts in the two premises. As a result, we can get each of them alone at lines in the derivation by means of applying ampersand elimination to those two lines as follows:

1. (C <-> D) & F	P
2. G & (H v I)	P
3. F	&E: 1
4. G	&E: 2

And now that the two conjuncts of the conjunction we are trying to derive each occur alone at a line, we can use ampersand introduction to introduce this conjunction at a new line, thereby completing the derivation:

1. (C <-> D) & F	P
2. G & (H v I)	P
3. F	&E: 1
4. G	&E: 2
5. F & G	&I: 3, 4

Note: when a rule is applied to statements at multiple lines, you need to list the line number at which each of these statements occur.

According to **vee introduction** (vI),

Vee Introduction (vI)

<u>p</u>	*- OR -*	*<u>p</u>*
p v q		*q v p*

if we have a statement of any kind at a line in our derivation, we can introduce the disjunction of that statement and *any other of symbolic logic we might like* at a new line in the derivation. Suppose we wanted to complete a derivation for the following argument:

A & (B -> C)
(D <-> E) v (B -> C)

We would begin the derivation as usual:

1. A & (B -> C)	P

The first thing to note is that one of the disjuncts of target disjunction happens to be one of the conjuncts of the premise at line 1. As a result, we can use &E to get this statement alone at a line as follows:

1. A & (B -> C)	P
2. B -> C	&E: 1

Now we are in a position to use vee introduction to derive our target disjunction:

1. A & (B -> C)	P
2. B -> C	&E: 1
3. (D <-> E) v (B -> C)	vI: 2

Note: although vee introduction is an easy rule to use, it has to be deployed strategically. You need to decide what disjunction it would be useful to have as a line in your derivation and then try to use addition to derive that disjunction in particular.

According to **arrow elimination** (->E),

Arrow Elimination (->E)
p -> q
p
q

if we have a conditional statement at a line in our derivation and we have the antecedent of that conditional at another line in our derivation, we can introduce the consequent of the conditional at a new line in the derivation. Suppose we want to construct a derivation for the following argument:

(A v B) ⊃ ~C
D & B
~C

We begin our derivation as follows:

1. (A v B) ⊃ ~C	P

2. D & B	P

The first thing to note is that the premise at line 1 of the derivation is of the form "*p* ⊃ *q*", where "(A v B)" is substituted in place of "*p*" and "~C" is substituted in place of "*q*". As a result, in order to use ampersand elimination to derive the consequent of this conditional, "~C", we will have to get the antecedent, "A v B" alone at a line. And since this statement is a disjunction, we can do this by first, deriving one its disjuncts and second, then applying vee introduction as follows:

1. (A v B) ⊃ ~C	P
2. D & B	P
3. B	&E: 2
4. A v B	vI: 3

Note: although we could have used vee introduction to derive a disjunction whose disjuncts were "B" and any symbolic statement whatsoever, we strategically used this rule to derive the antecedent of the conditional at line 1 in particular. We are now in a position to complete our derivation as follows:

1. (A v B) ⊃ ~C	P
2. D & B	P
3. B	&E: 2
4. A v B	vI: 3
5. ~C	->E: 1, 4

Note that the justification for arrow elimination includes two line numbers: the number of the line at which the conditional occurs and the number of the line at which the antecedent of the conditional occurs.

According to **vee elimination** (vE),

Vee Elimination (vE)

(p -> r) & (q -> r)
<u>*p v q*</u>
r

if we have a conjunction of conditional statements *with the same consequent* at a line in our derivation and the disjunction of the antecedents of the two conditionals at another line in our derivation, we can introduce the consequent of the two conditionals at a new line in the derivation.

Suppose we want to construct a derivation for the following argument:

(A v B) & (B -> ~D)
<u>A -> ~D</u>
~D

We would begin the derivation as follows:

1. (A v B) & (B -> ~D)	P
2. A -> ~D	P

The first thing to note is that the first conjunct of the conjunction at line 1 is a disjunction and, moreover, its disjuncts are the antecedents of the conditionals at line 1 (the second conjunct) and line 2. As a result, our strategy is going to consist in reorganizing these statements so that the disjunction occurs alone at a line in the derivation and the two conditionals occur as the conjuncts of a conjunction at another line. To this end, we will have to first apply ampersand elimination twice to line 1 as follows:

1. (A v B) & (B -> ~D)	P
2. A -> ~D	P
3. A v B	&E: 1
4. B -> ~D	&E: 1

The next step in the derivation involves introducing a conjunction of the conditionals at lines 2 and 4 by means of the application of ampersand introduction as follows:

1. (A v B) & (B -> ~D)	P
2. A -> ~D	P
3. A v B	&E: 1
4. B -> ~D	&E: 1
5. (A -> ~D) & (B -> ~D)	&I: 2, 4

Now that we have the disjunction “A v B” alone at a line and the requisite conjunction of conditionals at another line, we can use vee elimination to complete the derivation:

1. (A v B) & (B -> ~D)	P
2. A -> ~D	P
3. A v B	&E: 1
4. B -> ~D	&E: 1
5. (A -> ~D) & (B -> ~D)	&I: 2, 4
6. ~D	vE: 3, 5

Note that in the justification for vee elimination we need to need to include the line number where the disjunction occurs and the line number where the conjunction of conditionals occurs.

According to **double-arrow introduction** (<->I),

Double-Arrow Introduction (<->I)
(p -> q) & (q -> p)
p <-> q

if we have a conjunction of conditionals at a line in a derivation in which (i) the antecedent of the first is the consequent of the second and (ii) the consequent of the first is the antecedent of the second, we can introduce a biconditional at a new line with the antecedent and consequent of the first conditional on either side of the double arrow. Suppose we want to construct a derivation for the following argument:

(A -> B) & ~C
~C -> (B -> A)
A <-> B

We would begin the derivation as usual:

1. (A -> B) & ~C	P
2. ~C -> (B -> A)	P

The first thing to note is that the first conjunct at line 1 and the consequent of the conditional at line 2 are both conditionals but with their antecedents and consequents reversed. Our strategy is going to involve, first, getting each of these conditionals alone at a line in our derivation and, second, getting them to occur as the conjuncts of a conjunction at a subsequent line. The first step in our derivation consists in applying ampersand elimination to line 1 as follows:

1. (A -> B) & ~C	P
2. ~C -> (B -> A)	P
3. A -> B	&E: 1

Next we will derive the consequent of the conditional at line 2 by deriving the antecedent and then applying the rule arrow elimination:

1. (A -> B) & ~C	P
2. ~C -> (B -> A)	P
3. A -> B	&E: 1
4. ~C	&E: 1
5. B -> A	->E: 2, 4

We can complete our derivation by forming the conjunction of the conditionals at lines 3 and 5 using ampersand introduction and then applying double-arrow introduction to the result:

1. (A -> B) & ~C	P
2. ~C -> (B -> A)	P
3. A -> B	&E: 1
4. ~C	&E: 1
5. B -> A	->E: 2, 4
6. (A -> B) & (B -> A)	&I: 3, 5
7. A <-> B	<->I: 6

Finally, according to **double-arrow elimination** (<->E),

Double-Arrow Elimination (<->E)
p <-> q
(p -> q) & (q -> p)

if we have biconditional at a line in a derivation, we can introduce a conjunction of conditionals at a new line in which (i) the first component of the biconditional is the antecedent of the first conditional and the consequent of the second and (ii) the second component of the biconditional is the consequent of the first conditional and the antecedent of the second. Suppose we want to construct a derivation for the following argument:

A <-> (B v C)
B
A

We would set up our derivation as follows:

1. A <-> (B v C)	P
2. B	P

Given that statement at line 1 is a biconditional, a natural first step would be to apply biconditional elimination to it as follows:

1. A <-> (B v C)	P
2. B	P
3. [A -> (B v C)] & [(B v C) -> A]	<->E: 1

Since the conclusion of the argument we are considering is the consequent of the conditional that is the second conjunct at line 3, a good strategy would be to get this conditional alone at a line and then use arrow elimination to derive its consequent. To this end, our next step would be the following:

1. A <-> (B v C)	P
2. B	P

3. [A -> (B v C)] & [(B v C) -> A]	<->E: 1
4. (B v C) -> A	&E: 3

Before we can use arrow elimination to derive our final conclusion, we need the antecedent of the conditional at line 4, “B v C”, at a separate line. We can easily derive this, however, by applying vee introduction to line 2 as follows:

1. A <-> (B v C)	P
2. B	P
3. [A -> (B v C)] & [(B v C) -> A]	<->E: 1
4. (B v C) -> A	&E: 3
5. B v C	vI: 2

Now we are in a position to complete our derivation using arrow elimination as follows:

1. A <-> (B v C)	P
2. B	P
3. [A -> (B v C)] & [(B v C) -> A]	<->E: 1
4. (B v C) -> A	&E: 3
5. B v C	vI: 2
6. A	->E: 4, 5

B. Sub-derivation Rules

We will now consider inference rules that involve the use of sub-derivations, that is, derivations within derivations. When you use a sub-derivation rule, you make an assumption and then attempt to derive a statement or statements from this assumption and other statements that previously occurred in the main derivation. On the basis of what you derive in your sub-derivation, a sub-derivation rule will permit you to derive a statement in the main derivation. Sub-derivation rules have to be used strategically – with a careful choice of what statements will be assumed and derived – in order to bear fruit in the main derivation. *And it is important to note that once a sub-derivation has been completed, none of the statements that occur in it can be used in subsequent steps in the derivation*.

We will be looking at the following three sub-derivation rules in this section:

Arrow Introduction (->I)

```
      p            A
      q
p -> q
```

Tilde Introduction (~I)

```
      p            A
      q & ~q
~p
```

Tilde Elimination (~E)

~p   *A*
q & ~q
p

According to **arrow introduction** (->I),

Arrow Introduction (->I)

p   *A*
q
p -> q

if you assume a statement of our symbolic language and derive a statement using that assumption together with any other prior lines in the derivation (except those that occur in completed sub-derivations), then you can derive a conditional in the main derivation whose antecedent is the statement you assume and whose antecedent is the statement you derive (in the sub-derivation). Suppose, for example, that we want to construct a derivation for the following argument:

A -> B
(B v C) -> D
A -> D

We would begin by setting up our derivation in the ordinary way:

1. A -> B   P
2. (B v C) -> D   P

Since our goal is to derive a conditional, we would normally use arrow introduction to do so. This requires that we use a sub-derivation in which we assume the antecedent of our target conditional – "A" – and attempt to derive the consequent of that conditional – "D". We begin as follows:

1. A -> B   P
2. (B v C) -> D   P
  3. A   A

Note: our justification for line 3 is "A" which abbreviates "assumption." Since the statements at lines 1 and 2 do not occur in completed sub-derivations, we can use them together with our assumption at line 3 in order to derive the consequent of our target conditional, "D". The first thing to note is that the statement at line 3 is the antecedent of the conditional at line 1. This means we can use arrow elimination to derive the consequent of the conditional at line 1:

1. A -> B   P
2. (B v C) -> D   P
  3. A   A

4. B	->E: 1, 3

Since the statement at line 4 is one disjunct of the antecedent of the conditional at line 2, we can use vee introduction to get the antecedent of this conditional and, then, arrow elimination to derive its consequent:

1. A -> B	P
2. (B v C) -> D	P
3. A	A
4. B	->E: 1, 3
5. B v C	vI: 4
6. D	->E: 2, 5

Finally, since we assumed the antecedent of the conclusion of the argument at issue and derived its consequent, we can now introduce this conditional statement in the main derivation as follows:

1. A -> B	P
2. (B v C) -> D	P
3. A	A
4. B	->E: 1, 3
5. B v C	vI: 4
6. D	->E: 2, 5
7. A -> D	->I: 3-6

When justifying a line in a derivation by appeal to a sub-derivation rule, we indicate a range of line numbers beginning with the line at which the assumption is made and ending with the line at which the sub-derivation terminates.

According to **tilde introduction** (~I),

Tilde Introduction (~I)

p	*A*
q & ~q	
~p	

if you assume a symbolic statement and derive a contradiction – a conjunction whose conjuncts are a statement and its negation – using that assumption together with any other prior lines in the derivation (except those that occur in completed sub-derivations), then you can derive the negation of your assumption in the main derivation. Suppose, for example, that we want to construct a derivation for the following argument:

(A v D) -> (F & B)
~B
~(A v D)

We would start our derivation in the usual way:

1. (A v D) -> (F & B)	P
2. ~B	P

Since our target statement – "~(A v D)" – is a negation, and there is no obvious alternative strategy, a natural suggestion would be to attempt to derive it using tilde introduction. In order to do so, we must set up a sub-derivation in which we assume the unnegated "A v D" and attempt to derive a contradiction. We begin as follows:

1. (A v D) -> (F & B)	P
2. ~B	P
3. A v D	A

Since our assumption at line 3 is the antecedent of the conditional at line 1, we can now use arrow introduction to derive the consequent of this conditional:

1. (A v D) -> (F & B)	P
2. ~B	P
3. A v D	A
4. F & B	->E: 1, 3

Notice that the statement at line 2 is the negation of the second conjunct of the statement at line 4. This suggests that we should try to derive "B & ~B" as our contradiction in the sub-derivation. We can easily do this in two steps as follows:

1. (A v D) -> (F & B)	P
2. ~B	P
3. A v D	A
4. F & B	->E: 1, 3
5. B	&E: 4
6. B & ~B	&I: 2, 5

Finally, since we have derived a contradiction within our sub-derivation, we can introduce the negation of the assumption at line 3 in the main derivation, thereby completing the derivation, as follows:

1. (A v D) -> (F & B)	P
2. ~B	P
3. A v D	A
4. F & B	->E: 1, 3
5. B	&E: 4
6. B & ~B	&I: 2, 5
7. ~(A v D)	~I: 3-6

As with the previous case, the justification for line 7 includes the range of numbers from the line at which the assumption is made to the line at which the contradiction is derived.

Finally, according to tilde elimination (~E),

Tilde Elimination (~E)

	~p	*A*
	q & ~q	
p		

if you assume a negated statement and derive a contradiction using that assumption together with any other prior lines in the derivation (except those that occur in completed sub-derivations), then you can derive the unnegated variant of your assumption in the main derivation. Suppose, for example, that we want to construct a derivation for the following argument:

(A v B) -> C
<u>(~B -> D) & ~D</u>
C

We start our derivation as usual:

1. (A v B) -> C	P
2. (~B -> D) & ~D	P

The first thing to note is that the conclusion of the argument is the consequent of the conditional at line. As a result, a good strategy would be to derive the antecedent of this conclusion and then use arrow elimination to derive the consequent. In order to derive the antecedent, "A v B", we could derive one of the conjuncts by itself and then use vee introduction. Since "A" appears nowhere else in the argument but "~B" does, our best bet would be to attempt to derive "B" using tilde elimination. To this end, we set up our sub-derivation as follows:

1. (A v B) -> C	P
2. (~B -> D) & ~D	P
3. ~B	A

Since "~B" is the antecedent of the conditional that is the first conjunct at line 2, one natural approach would be to get this conditional by itself at a line using ampersand elimination and then apply arrow elimination as follows:

1. (A v B) -> C	P
2. (~B -> D) & ~D	P
3. ~B	A
4. ~B -> D	&E: 2
5. D	->E: 3, 4

Given that second conjunct of the conjunction at line 2 is the negation of the statement at line 5, we can easily derive our required contradiction in the sub-derivation as follows:

1. (A v B) -> C	P
2. (~B -> D) & ~D	P
3. ~B	A
4. ~B -> D	&E: 2
5. D	->E: 3, 4
6. ~D	&E: 2
7. D & ~D	&I: 5, 6

Next, since we have derived a contradiction within our sub-derivation, we can introduce the unnegated assumption into the main derivation:

1. (A v B) -> C	P
2. (~B -> D) & ~D	P
3. ~B	A
4. ~B -> D	&E: 2
5. D	->E: 3, 4
6. ~D	&E: 2
7. D & ~D	&I: 5, 6
8. B	~E: 3-7

Finally, we can complete the derivation using vee introduction and arrow elimination as follows:

1. (A v B) -> C	P
2. (~B -> D) & ~D	P
3. ~B	A
4. ~B -> D	&E: 2
5. D	->E: 3, 4
6. ~D	&E: 2
7. D & ~D	&I: 5, 6
8. B	~E: 3-7
9. A v B	vI: 8
10. C	->E: 1, 9

Exercises Section 7.2

I. *Use the eight basic inference rules to derive the conclusion of each of the following arguments. Be sure to include a justification for each line in your derivations.*

1. A -> (B & C)
   <u>D & A</u>

C & D

2. (A v B) -> C
B & D
C & D

3. ((A & B) & C) & D
B v (E <-> F)

4. B
[(B & B) v F] & [(B & B) v F]

5. (A & B) & (C & D)
[(B v E) & (C v F)] -> G
G & A

6. (A v B) & (B -> C)
(B -> C) -> (A -> C)
C

7. A -> (B -> (C -> D))
(A & B) & C
(D v E) & A

8. (A -> B) & (C -> (B -> A))
(C v D) & (D -> (B -> A))
A <-> B

9. A <-> B
B <-> A

10. (A -> C) <-> B
B & (C -> A)
A <-> C

11. A -> (B & C)
(B -> D) & A
C & D

12. A <-> B
B <-> C
A v C
B

13. A -> (B -> (C -> D))
B -> (A -> (D -> C))
A & B
D <-> C

14. A <-> B
[(A -> B) -> (D -> C)] & [(B -> A) -> (C -> D)]
C <-> D

15. A <-> (B -> C)
A & (D v E)
D -> (C -> B)
A -> [E -> (C -> B)]
B <-> C

II. *Use the eight basic inference rules and three sub-derivation rules to derive the conclusion of each of the following arguments. Be sure to include a justification for each line in your derivations.*

1. A -> B
B -> C
A -> C

2. A -> B
~B
~A

3. A -> (B & C)
C -> A
A <-> C

4. (A v ~C) -> B
(C -> D) & ~B
D

5. A -> (B & ~C)
B -> C
~A

6. B
A -> (C -> (D -> B))

7. A -> ~B
~B -> ~A
C -> A
~C

8. A <-> B
C -> (D -> B)
D v A
C -> B

9. A -> ~B
B
~A -> D
C -> (D v E)

10. ~A v B
A -> B

11. ~(A v B)
~A & ~B

12. A & B
A <-> B

13. A v B
~B
A

14. A v B
A -> C
B -> D
C v D

15. A <-> B
B <-> C
A <-> C

16. A <-> B
(C -> B) & (~C -> ~B)
C <-> A

3. Derivation Strategies

Completing a derivation can sometimes be daunting. When first learning derivations, students often feel that they are simply using rules randomly without any clear strategy for deriving the conclusion. But although there is no strict formula for completing derivations, there are nevertheless certain rules of thumb that can make things a lot easier.

Step 1: Locate your target statement

The first thing you need to do is to is ask the following questions about the statement you are trying to derive:

A. Does it appear anywhere in the premises?

The question is whether the statement you are trying to derive appears in the premises (or other statements you have already derived) in particular as a conjunct of a conjunction, as the consequent of a conditional, or as a component of a biconditional. If your answer to this question is yes, you go to step 2; if your answer is no, you go to step 3.

Step 2: Extracting your target statement

If your target statement appears in your premises, you need to figure out two things: what, if any, rule(s) you can use to extract it from the premises; and what if any other statements you need to derive in order to use the rule(s) in question. Suppose, for example, the conclusion of your argument is the following:

A -> B

And suppose that this statement occurs in the premises as one conjunct of a conjunction:

(A -> B) & (C v D)

In this case you could extract the conclusion from the premise using the rule ampersand elimination (&E), without needing any other statement to do so.

Now suppose that the conclusion of your argument occurs in the premises as the consequent of a conditional:

(C v D) -> (A -> B).

In this case you could extract the consequent using the rule arrow elimination (->E). But in order to do so you would need to already have alone at a line – or to derive – the antecedent of the conditional:

C v D

You would then go back to step 1 with this statement as your new target.

Finally, suppose that the conclusion of your argument occurs in the premises as a component of a biconditional:

(C v D) <-> (A -> B)

This case is the most complicated. You would, first, have to use double-arrow elimination (<->E) and, second, use ampersand elimination (&E) in order derive a conditional with your target statement as the consequent of a conditional:

(C v D) <-> (A -> B)
1. [(C v D) -> (A -> B)] & [(A -> B) -> (C v D)]
2. (C v D) -> (A -> B)

And then you could use arrow elimination to extract the conclusion from 2 as before.

Step 3: Deriving your target statement directly

If your target statement does not appear in your premises in a statement from which it can be easily extracted, then you will have to derive it directly. And how you go about doing so depends on what type of statement your target is: a negation, a conjunction, a disjunction, a conditional, or a biconditional. Given that we have introduction rules for each of the operators, there are natural derivation strategies worth considering for each statement type. We can tabulate this as follows:

Statement Type	Rules
Negation	Tilde Introduction (~I)
Conjunction	Ampersand Introduction (&I)
Disjunction	Vee Introduction (vI)
Conditional	Arrow Introduction (->I)
Biconditional	Double-Arrow Introduction (<->I)

Once you determine what rule you want to use to derive your target statement, you then need to determine what other statements you would need to have among your premises – or at other lines in your derivation – to be able to use that rule in that way. Suppose, for example, the conclusion of your argument is the following:

A <-> B

Now since your target statement is a biconditional you could try to derive it using double-arrow introduction. But in order to do so directly, you would need to have the following statements at lines in your derivation:

A -> B

and

B -> A

And if you do not already have one or the other of these statements alone at different lines in your derivation, you need to go back to step 1 with the missing statement(s) as your new target(s).

Chapter 8: Truth-tables

1. Characteristic Truth-tables

Propositional logic is truth functional in the sense that the truth-values – the truth or falsity – of compound statements are wholly determined by the truth-values of their component statements: if you know whether or not the components of a compound proposition are true or false, you can use the definitions of the logical operators to calculate the truth-value of the compound statement. Even though all compound statements of propositional logic are truth functional, the same is not true of all compound statements of English. Consider the following pair of propositions:

Global climate change is occurring

Fred believes that global climate change is occurring

The first proposition is a component of the second, but the truth of the first does not determine the truth of the second. After all, despite the fact that it is true that global climate change is occurring, Fred might simply refuse to believe it.

The definitions or meanings of the operators of propositional logic are given by their characteristic truth-tables, tables which show how the truth-values of statements containing those operators depend on the truth-values of their components. For the sake of generality, these definitions have to be formulated using statement variables rather than particular statements. **Statement variables** are expressions that can stand for any symbolic statement. In propositional logic we use lower case letters – "*p*," "*q*," "*r*," and "*s*" – as variables. Just as the variable "x" in mathematics can be used to represent any number, the variable "*p*" in propositional logic can be used to represent any statement, whether simple or compound.

Let us turn now to the characteristic truth-tables for the operators of propositional logic.

The truth-table for the tilde is as follows:

p	***~p***
T	**F**
F	**T**

What this tells us is that if a statement of the form *p* is true then its negation ~*p* is false; and if a statement of the form *p* is false then its negation ~*p* is true. Consider, for example, the following pair of a statement and its propositional logic translation:

The planet Earth is not flat ~E

Since “E” – which translates “The planet Earth is flat” – is false, the truth-table for the tilde tells us its negation “~E” is true, which is what we would expect.

The characteristic truth-table for the ampersand is as follows:

p	***q***	***p* & *q***
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**

This table tells us that a conjunction is true when both of its conjuncts are true but false otherwise. Consider, for example, the following pair of a statement and its propositional logic translation:

Triangles have three sides and snow is black T **&** B

Because the second conjunct is false, this truth-table tells us the whole conjunction is false as well.

The characteristic truth-table for the vee is as follows:

p	***q***	***p* v *q***
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

This truth-table tells us that a disjunction is true whenever at least one of its disjuncts is true and false only when both disjunct are false. Consider, for example, the following pair of a statement and its propositional logic translation:

Triangles have three sides or snow is black T **v** B

Because the first disjunct is true, this truth-table tells us the whole disjunction is true as well, despite the falsity of the second disjunct. It is worth noting that this truth-table captures the inclusive sense of disjunction because it entails that a disjunction is true when both disjuncts are true. Some sentences of English we might want to translate with the vee, however, are exclusive disjunctions in the sense that they are false when both disjuncts are true. A familiar example might be the following:

You can have either fries or a salad with your burger.

In propositional logic, we translate the exclusive disjunction as,

(F v S) **&** ~(F **&** S)

which means that you can have either fries or a salad but not both.

The characteristic truth-table for the arrow is as follows:

p	***q***	***p -> q***
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

This truth-table tells us that a conditional is false when the antecedent is true and the consequent is false but true otherwise. Consider, for example, the following pair of a statement and its propositional logic translation:

If Triangles have three sides then snow is black　　　　T -> B

Since the antecedent of this condition is true and the consequent is false, this truth-table tells us the whole conditional is false. It is worth noting that although the first two rows of the truth-table for the arrow are what you probably expected, some people find the last two rows surprising. Although there are some reasons for constructing the truth-table in this way, one lesson here is that the arrow is not an exact translation of the English “if … then … .”

The characteristic truth-table for the double arrow is as follows:

p	***q***	***p <-> q***
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

This truth-table tells us that a biconditional is true when both components have the same truth-value – they are both true or both false – and that it is false when the components differ in truth-value. Consider, for example, the following pair of a statement and its propositional logic translation:

Triangles have four sides if and only if snow is black　　　　T <-> B

Since both components of the biconditional are false, this truth-table tells us the whole biconditional is true.

2. Computing Truth-values

Because compound statements of propositional logic are truth functional, we can determine their truth-values using the truth-values of the simple component propositions together with the characteristic truth-tables for the logical operators we have been investigating. Consider the following compound proposition:

~(A & B) -> (C v ~D)

And let's suppose that the simple statements have the following truth-values:

A: T
B: F
C: F
D: T

The first step in calculating to the truth-value of the compound statement is to replace the simple propositions with their truth-values and write the resulting formula beneath the original:

~(A & B) -> (C v ~D)
~(***T*** & ***F***) -> (***F*** v ~***T***)

The next stage involves using the characteristic truth-tables and the truth-values we have calculated already to work out the truth-value of the compound. In general, we do our calculations in the following order:

1. Individual letters
2. Tildes preceding individual letters
3. Operators joining (negated or unnegated) letters
4. Tildes preceding parentheses

As we have already done calculation (1), we can now move to calculations in category (2). Using the first row of the characteristic truth-table for the tilde,

p	***~p***
T	**F**
F	**T**

yields the following:

~(A & B) -> (C v ~D)
~(T & F) -> (F v ~T)
~(T & F) -> (F v ***F***)

To do calculations in category (3), we first use the second row of the characteristic truth-table for the dot

p	***q***	***p* & *q***
T	**T**	**T**
<u>T</u>	**<u>F</u>**	**<u>F</u>**
F	**T**	**F**
F	**F**	**F**

which yields

~(A & B) -> (C v ~D)
~(T & F) -> (F v ~T)
~(T & F) -> (F v F)
~ ***F*** -> (F v F)

And second, we use the fourth row of the truth-table for the vee

p	***q***	***p* v *q***
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
<u>F</u>	**<u>F</u>**	**<u>F</u>**

which yields

~(A & B) -> (C v ~D)
~(T & F) -> (F v ~T)
~(T & F) -> (F v F)
~ F -> (F v F)
~ F -> ***F***

We can now use the second row of the truth-table for the tilde antecedent of the conditional we have been left with

p	***~p***
T	**F**
<u>F</u>	**<u>T</u>**

which yields

~(A & B) -> (C v ~D)
~(T & F) -> (F v ~T)
~(T & F) -> (F v F)
~ F -> (F v F)
~ F -> F
T -> F

Finally, we use the second row of the truth-table for the arrow

p	***q***	***p -> q***
T	**T**	**T**
<u>T</u>	**<u>F</u>**	**<u>F</u>**
F	**T**	**T**
F	**F**	**T**

which yields

~(A & B) -> (C v ~D)
~(T & F) -> (F v ~T)
~(T & F) -> (F v F)
~ F -> (F v F)
~ F -> F
T -> F
F

What we have shown is that the statement "~(A & B) -> (C v ~D)" is false when its simple components have the truth-values specified above.

Exercises Section 8.2

Using the indicated assignment of truth-values to the simple statements, determine the truth-value of each of the following compound statements.

A: F; B: T; C: T; D: F

1. ~A -> (B v A)

2. ~(A -> ~C)

3. (A & ~B) v (~A & B)

4. (C <-> D) <-> (D & C)

5. ~(A & ~(B v C))

6. (A <-> ~D) -> [(C v B) & ~A]

3. Truth-tables

A. Creating Truth-Tables

In this section we are going to learn how to create truth-tables. A **truth-table** is an arrangement of truth-values that shows, in every possible case, how the truth-value of a compound proposition is determined by the truth-values of its simple components. The first thing to determine is how many rows you are going to have in your truth table, that is, how many possible combinations of truth-values there are for the atomic statements that appear in the proposition or propositions you are assessing. If only one simple statement occurs in those propositions, then there are only two possible combinations of truth-values – the simple statement can be either true or false – and, hence, the truth-table will have two rows. If, however, there are two simple statements, there will be four combinations of truth-values and four rows in the truth table. And if there are three simple statements there will be eight combinations of truth-values and eight rows. And so on.

Suppose we were to produce a truth-table for the following proposition:

(~A & ~B) -> ~(B v ~A)

Since the proposition contains two simple statements, our truth-table will contain four rows. We write the simple statements to the left of the dividing line *in the order in which they occur in the proposition(s) being assessed*; and we write the propositions being assessed to the right of the dividing line, as below:

A	B	(~A & ~B)	->	~(B v ~A)

Next, we fill in the truth-values for the simple statements. We start with the statement immediately to the left of the dividing line and put in alternating T's and F's, as below:

A	B	(~A	&	~B)	->	~(B	v	~A)
	T							
	F							
	T							
	F							

We continue to the left by doubling the number of alternating T's and F's, as below.

A	B	(~A	&	~B)	->	~(B	v	~A)
T	T							
T	F							
F	T							
F	F							

Next we compute the truth-values for the larger components of the proposition, going from the smaller components to larger components, as we did in the previous section. The first step involves determining the truth-values for the negations of simple statements on each row (or truth-value assignment).

A	B	(~A	&	~B)	->	~(B	v	~A)
T	T	F		F				F
T	F	F		T				F
F	T	T		F				T
F	F	T		T				T

We then calculate the truth-values for the disjunction that is a component of the consequent of the consequent of the conditional on each row, using the characteristic truth-table for the vee together with the truth vales for B and ~A.

A	B	(~A	&	~B)	->	~(B	v	~A)
T	T	F		F			T	F
T	F	F		T			F	F
F	T	T		F			T	T
F	F	T		T			T	T

We then calculate the truth-values for the antecedent and the consequent of the conditional on each row, using the truth-tables for the ampersand and the tilde respectively.

A	B	(~A	&	~B)	->	~(B	v	~A)
T	T	F	F	F		F	T	F
T	F	F	F	T		T	F	F

F	T	T	F	F		F	T	T
F	F	T	T	T		F	T	T

Finally, we calculate the truth-values for the whole truth-table using the characteristic truth-table for the arrow, together with the truth-values of the antecedent and the consequent.

A	**B**	**(~A**	**&**	**~B)**	**->**	**~(B**	**v**	**~A)**
T	T	F	F	F	T	F	T	F
T	F	F	F	T	T	T	F	F
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	T	T

In your calculations, it is important to keep track of which column(s) you are using at each stage of your calculation. You can do this by drawing an arrow under the relevant column(s). You should also circle the column(s) corresponding to the truth-value for the target statement(s).

A	**B**	**(~A**	**&**	**~B)**	**->**	**~(B**	**v**	**~A)**
T	T	F	F	F	T	F	T	F
T	F	F	F	T	T	T	F	F
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	T	T

In creating truth-tables, you do not need to fill in every column under every operator in the target proposition(s): you will only be graded on the column under the main connective. But especially as you are starting out, it is a good idea to fill in most of the additional columns as well, in order to avoid making careless mistakes.

B. Individual Propositions

There are a number of different things we can use truth-tables to ascertain, including classifying individual propositions, comparing pairs (or larger groups) of propositions, and evaluating arguments. In this section, we will learn how to use truth-tables to classify individual propositions. In particular, we will classify compound propositions in terms of how their truth-values vary with the truth-values of their simple components. A compound statement is **logically true** if and only if true on every truth-value assignment, regardless of the truth-values of its simple components. A statement is **logically false** if and only if it is false on every truth-value assignment, regardless of the truth-values of its simple components. And a statement is **logically contingent** if and only if its truth-value varies with the truth values of its simple components and, so, is true on at least one truth-value assignment and false on at least truth-value assignment.

In order to determine whether a particular proposition is logically true, logically false, or logically contingent, we create a truth-table for the target proposition in exactly the same way we did in the previous section. And we interpret the resulting table by looking at the column under the main connective. If there are only T's in that column then the statement is logically true. If there are only Fs in that column then the statement is logically false. And if there is a mix of T's and F's then the statement is logically contingent. Consider again the truth-table we produced in the previous section:

A	B	(~A	&	~B)	->	~(B	v	~A)
T	T	F	F	F	T	F	T	F
T	F	F	F	T	T	T	F	F
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	T	T

Because under the main connective we find a mix of T's and F's, this truth-table establishes that the statement "(~A & ~B) -> ~(B v ~A)" is logically contingent. But consider now the table for the statement "(~A & ~B) -> ~(B v A)":

A	B	(~A	&	~B)	->	~(B	v	A)
T	T	F	F	F	T	F	T	
T	F	F	F	T	T	F	T	
F	T	T	F	F	T	F	T	
F	F	T	T	T	T	T	F	

Because under the main connective we find only T's, the truth-table establishes that the target statement is logically true.

C. Pairs/ Groups of Propositions

In this section we will learn how to compare pairs (or larger groups) of propositions. The question we will be focusing on is the extent to which propositions share truth-values on different truth-value assignments. Two (or more) propositions are **logically equivalent** if and only if they have the same truth-value on each truth-value assignment. Two (or more) propositions are **logically contradictory** if and only if they have opposite truth-values on each truth-value assignment. Two (or more) propositions are **logically consistent** if and only if there is at least one truth-value assignment on which they are both (or

all) true. And finally, two (or more) propositions are **logically inconsistent** if and only if there is no truth-value assignment on which they are both (or all) true.
In order to compare propositions, we create a truth-table in which both (or all) propositions appear. Suppose, for example, we want to compare the following pair of propositions:

(~A & ~B) -> ~(B v ~A)   (~A & ~B) -> ~(B v A)

We begin by creating a truth-table in which both propositions appear, as below.

A	**B**	**(~A & ~B) -> ~(B v ~A)**	**(~A & ~B) -> ~(B v A)**
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

As above, show as much of your work as you need to in order to calculate the truth-values for each statement under the main connective. Now since they have different truth-values on the fourth row/ truth-value assignment, they cannot be logically equivalent. But since they have the same truth-value on the first three rows/ truth-value assignments they cannot be contradictory. Finally, since they are both true on the first three truth-value assignments, it follows that there is at least one truth-value assignment on which they are both true; hence they are consistent. But it is important to note that the fact that there is one truth-value – the fourth – on which they differ in their truth-value does not show that they are inconsistent (nor would a truth-value assignment on which they were both false). In order to be inconsistent, there needs to be no truth-value assignment on which they are both true.

It is worth noting that if a pair of propositions is contradictory then they are automatically inconsistent: if they have opposite truth-values on every truth-value assignment, there will be no truth-value assignment on which they both have the value T. The opposite, however, is not true. In fact, if both propositions are logically false– that is, false on each truth-value assignment – they will be logically equivalent and inconsistent.

D. Truth-tables for Arguments

In this section, we will learn to use truth-tables to assess the validity of arguments. The basic definition of validity is as follows: an argument is valid if and only if it is not possible for the premises to be true and the conclusion false; an argument is invalid if and only if it is possible for the premises to be true and the conclusion false. The key to understanding how to use truth-tables to assess validity is to realize that the truth-value assignments – the various combination of truth-values of the simple statements – correspond to the possibilities in question. That is to say, each truth-value assignment – each row in a truth-table– is one of the possible combinations of truth and falsity that the premises and conclusion might have. As a result, if none of the truth-value assignments make the premises of an argument true and the conclusion false, then it is not possible for the premises to be true and the conclusion false. Hence, the argument is valid. But if just one of the truth-value assignments makes the premises true and the conclusion false, then it is possible for the premises to be true and the conclusion false. Hence the argument is invalid.

In order to assess the validity of an argument using truth-tables, we need to create a truth-table in which the premises and the conclusion all appear, with a single slash between the premises and the double slash between the last premise and the conclusion. Consider the following symbolic argument:

(A <-> B) & ~C
C v A
B v B

The truth-table for this argument looks as follows:

A	**B**	**C**	**(A <-> B) & ~C /**	**C v A //**	**B v B**
T	T	T	T F F	T	T
T	T	F	T T T	T	T
T	F	T	F F F	T	F
T	F	F	F F T	T	F
F	T	T	F F F	T	T
F	T	F	F F T	F	T
F	F	T	T F F	T	F
F	F	F	T T T	F	F

Now in order to use this truth-table to evaluate the validity of the argument, the first thing we need to do is to identify the truth-value assignments which make all of the premises true. You can indicate this with an arrow or by circling the truth-values of the premises (or both). Since the second truth-value assignment is the only one on which both premises are true, we can mark up our diagram as follows:

A	**B**	**C**	**(A <-> B) & ~C /**	**C v A //**	**B v B**
T	T	T	T F F	T	T
T	T	F	T T T	T	T
T	F	T	F F F	T	F
T	F	F	F F T	T	F
F	T	T	F F F	T	T
F	T	F	F F T	F	T
F	F	T	T F F	T	F
F	F	F	T T T	F	F

Finally, we look at the truth-values of the conclusion on every truth-value assignment which makes all of the premises true. If in *every* such case the conclusion is also true, then the argument is valid. But if in just *one* case the conclusion is false, the argument is invalid. In our example, the only truth-value assignment – the second row – that makes all the premises true also makes the conclusion true. As a result, the argument is valid. We can indicate this by putting a square around the truth-value of the conclusion.

A	**B**	**C**	**(A <-> B) & ~C /**	**C v A //**	**B v B**
T	T	T	T F F	T	T
T	T	F	T T T	T	T
T	F	T	F F F	T	F
T	F	F	F F T	T	F
F	T	T	F F F	T	T
F	T	F	F F T	F	T
F	F	T	T F F	T	F
F	F	F	T T T	F	F

There are a number of common mistakes that students make in interpreting truth-tables, including the following:

1. An argument is valid if there is one truth-value assignment on which all the premises are false and the conclusion true – INCORRECT
2. An argument is valid if there is one truth-value assignment on which the premises and conclusion are all true (even if there is another assignment on which the premises are true and the conclusion is false) – INCORRECT

Finally, it is worth noting that if the premises of an argument are inconsistent – there is no truth-value assignment on which they are all true – then it is automatically valid. After all, if there is no truth-value assignment on which all of the premises are true then there is no truth-value assignment on which all of the premises are true and the conclusion false.

Exercises Section 8.3

Using truth-tables, determine whether each of the following statements is logically true, logically false, or logically contingent.

1. (~A & A) -> ~(A v ~A)

2. (A <-> ~B) & (~A v ~B)

3. (A -> ~B) <-> (~A -> B)

4. ~A & (B v (C <-> A))

5. [~A -> (B -> ~C)] -> [(~A & B) -> ~C]

Using truth-tables, determine whether the members of each of the following pairs of statements are logically equivalent, logically contradictory, logically consistent, or logically inconsistent. Note: a pair of statements can have more than one of these features.

6. (~A v B)　　(A & ~B)

7. A -> (B -> ~A)　　(A & B) -> ~A

8. (~A v ~A) & ~B　　~(B -> ~A)

9. A <-> ~(B v C)　　[A -> ~B] & [~A -> C]

10. A -> ~(B <-> C)　~A v (C <-> B)

Using truth-tables, determine whether each of the following arguments is valid or invalid.

11. (A v B) -> ~B
A & ~B
~B v A

12. (A v ~B) <-> B
A v (B & ~B)
B -> ~A

13. (A <-> B) <-> A
~A v B
~(B v ~B)

14. A -> (~B -> A)
(A <-> C) & ~B
~(B <-> ~C) v B

15. (~A -> ~C) & (B -> ~C)
(~A v B) & ~B
~(B v C)

Answers to Exercises

Section 1.1

1. Non-statement - request

2. Statement – true

3. Non-statement – order

4. Statement – false

5. Non-statement – question

6. Statement – true

7. Non-statement – bet

8. Statement – false

9. Non-statement – promise

10. Non-statement – question

11. Non-statement – request

12. Statement – true

13. Statement – true?

14. Non-statement – order

15. Statement – false?

Section 1.2

1. deductive

2. inductive

3. deductive

4. deductive

5. inductive

6. inductive

7. deductive

8. inductive

9. deductive

10. deductive

11. inductive

12. deductive

13. deductive

14. inductive

15. inductive

Section 1.4

1. non-argument – statement of belief

2. Argument – C: we’re going to have a bland but healthy side dish

3. non-argument – loosely associated statements

4. non-argument – explanatory passage

5. non-argument – loosely associated statements

6. non-argument – illustrative passage

7. Argument – C: if it rains, water will seep into my basement.

8. non-argument – conditional statement

9. non-argument – expository passage

10. non-argument – report

11. non-argument – illustrative passage

12. non-argument – explanatory passage

13. non-argument – conditional statement

14. Argument – C: Fred will go to the party

15. non-argument – expository passage

Section 2.1

1. P_1: Public education is one of the most expensive items in the provincial budget
   P_2: The people who benefit most directly from it pay little or nothing by way of taxes
   C: Public schools should be turned into workhouses for poor children

2. P1: Our current crop of politicians is primarily motivated by greed
   P2: It's a good idea to change the party in power every election
   C: We need to vote them [our current crop of politicians] out in the next election

3. P_1: The current centers on the roster aren't strong enough to defend against their bigger counterparts on other teams
   P_2: Their likely draft position is too low to acquire an adequate big man through the draft
   C: The Raptors need to trade for a new starting center

4. P1: Johnny Depp has been proven to be guilty of spousal abuse
   P2: His contract demands are too high for his movies to be profitable
   C: Hollywood studios should refrain from hiring him [Depp] to star in their movies

5. P_1: People who refuse to get vaccinated pose a risk to the health of other people
   P_2: The fewer anti-vaxxers there are, the lower the risk they create
   C: People should be allowed to refuse to take vaccines for diseases with high mortality rates

6. P_1: Logic classes are boring
   P_2: Logic tests are too hard
   P_3: If the tests in a course are too hard, you'll end up with a bad grade
   C: You should drop logic.

7. P_1: You either have to get up early or you'll be late for class
   P_2: I hate getting up early
   P_3: Professors are always snarky when you're late
   C: Face-to-face courses are the worst

8. P_1: Any animal which demands too much of your attention makes a bad pet
   P_2: Cats are more independent than dogs

P_3: Their [cats'] excrement is not spread all over your yard
C: Cats make much better pets than dogs

9. P_1: Baths involve wallowing in your own filth
P_2: They [baths] use too much hot water
P_3: Reducing our energy use helps in the fight against global warming
C: Regular showering is the best way to keep clean

10. P_1: Bad guitar playing is really annoying
P_2: They only way to avoid it is to require guitar players to get licenses before playing in public or to ban public guitar playing altogether
P_3: Licensing guitar playing would involve government overreach
C: We need a complete ban on public guitar playing

11. P_1: People often can't make it home before they have to go
P_2: A lot of businesses won't let you use their washrooms unless you buy something first
P_3: Homeless people have no alternative to urinating in public
P_4: If people cannot see you go, they have no reason to complain
C: As long as they're discreet about it, people should be allowed to urinate wherever they like

12. P_1: If we defund the police, we're going to have to fire a lot of police officers and replace them with social workers
P_2: The result would be a smaller force of officers dealing with the same number of crimes
P_3: Overworked and demoralized employees are always less effective at their jobs
P_4: Social workers would not be able to pick up the slack
C: The police should not be defunded

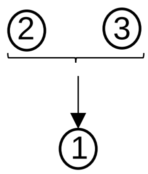
13. P_1: There are large numbers of COVID-denying anti-maskers in the US
P_2: They would have no reason not to come to Canada
P_3: If they did come, they would ignore our mask use and social distancing policies
P_4: There's no better way than that to cause our numbers to spike
C: If we re-open the border with the US, our COVID-19 case numbers will spike

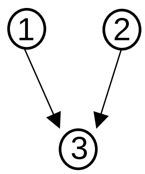
14. P_1: Social distancing cannot be maintained in U of S classrooms
P_2: There's plenty of room to spread out in the Bowl
P_3: There are no worries about ventilation
P_4: U of S students are used to the cold
C: We should hold our face-to-face classes in the Bowl

15. P1: We can either keep our stuff or send it to the landfill
P2: Burying our garbage is unsustainable
P3: Retail shopping is the backbone of the Canadian economy

P4: Cutting back on shopping would put people out of work
C: Hoarding should be encouraged

Section 2.2

1. 

2. 

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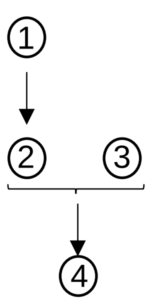
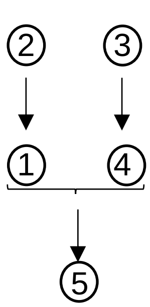
4. 

5. 

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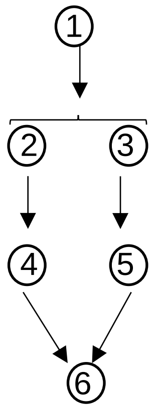

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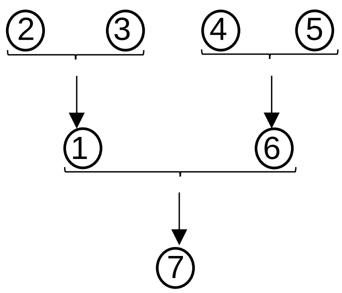

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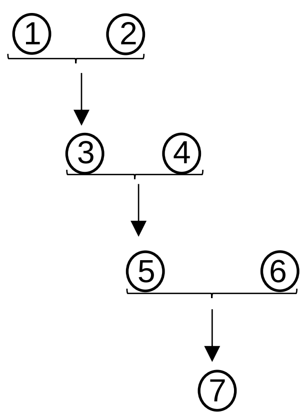

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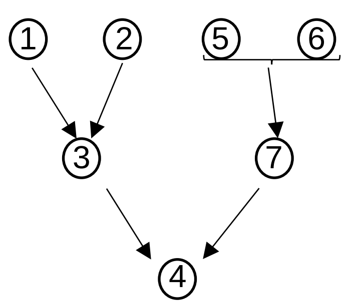

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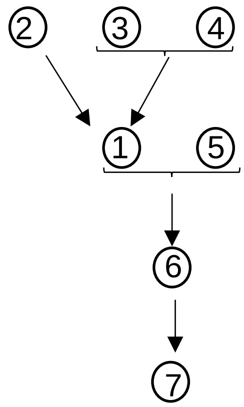

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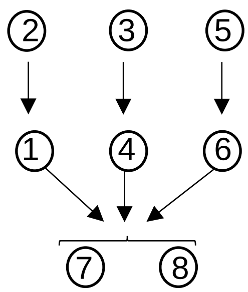

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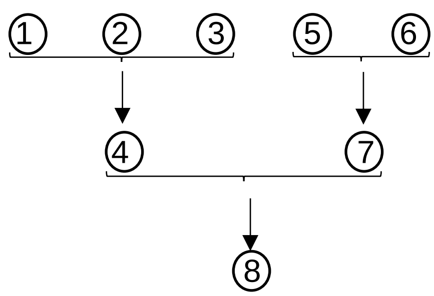

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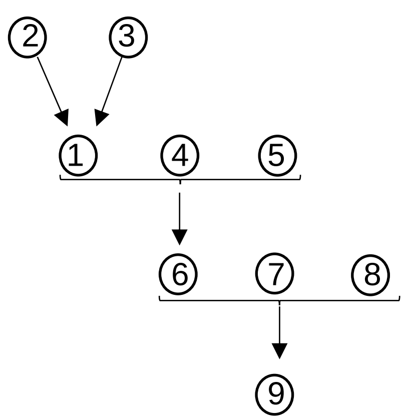

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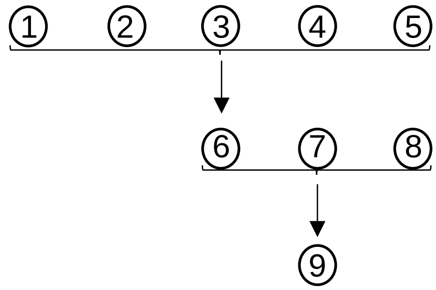

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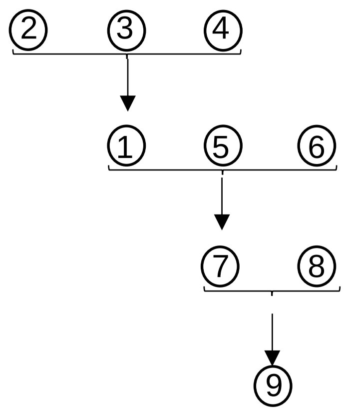


Section 3.1

1. ad hominem – abusive

2. red herring

3. ad populum

4. slippery slope

5. straw person

6. ad hominem – circumstantial

7. loaded question

8. composition

9. ad ignorantiam

10. ad baculum

11. false dilemma
12. accident
13. no fallacy
14. red herring
15. ad hominem – tu quoque
16. syntactic ambiguity
17. irrelevant conclusion
18. hasty generalization
19. straw person
20. equivocation
21. ad misericordiam
22. slippery slope
23. false dilemma
24. ad hominem – circumstantial
25. division
26. ad ignorantiam
27. ad populum
28. no fallacy
29. accident
30. irrelevant conclusion

Section 4.1

1. *Quantifier*: All

Copula: are
Subject Term: things that go bump in the night
Predicate Term: monsters hiding in my closet

2. *Quantifier*: Some
Copula: are not
Subject Term: very important people
Predicate Term: purveyors of depth or substance

3. *Quantifier*: Some
Copula: are
Subject Term: classes taught at U of S
Predicate Term: hotbeds of pestilence and disease

4. *Quantifier*: No
Copula: are
Subject Term: objects I have in my pocket
Predicate Term: invisibility rings

5. *Quantifier*: Some
Copula: are not
Subject Term: members of outlaw motorcycle gangs
Predicate Term: people I would bring home to meet my parents

6. *Quantifier*: All
Copula: are
Subject Term: spicy curries
Predicate Term: meals you should not serve to toddlers

7. *Quantifier*: Some
Copula: are
Subject Term: people who are not worthy of a Nobel Prize
Predicate Term: people who are worthy of less prestigious prizes

8. *Quantifier*: No
Copula: are
Subject Term: socks without holes in them
Predicate Term: clothes in need of darning

9. *Quantifier*: All
Copula: are
Subject Term: people who drink too much and talk to loud
Predicate Term: inappropriate tearoom patrons

10. *Quantifier*: Some
Copula: are not
Subject Term: questions on the multiple-choice section of the final exam

Predicate Term: questions that are entirely unfair

Section 4.2

1. Positive

2. Negative

3. Positive

4. Negative

5. Negative

6. Universal

7. Particular

8. Particular

9. Universal

10. Universal

Section 4.3

1. Some seven-footers are not good basketball players.

2. Some rescue dogs are animals that are unhappy in their new homes.

3. No places in Regina are places good live music can be found.

4. All piano teachers are people who eventually lose interest in the instrument.

5. All people who sing the blues are cowboys.

6. Some squirrels with red fur and bushy tails are animals nesting in the crawl space in behind my closet.

7. All trees that fall alone in the forest are things that do not make a sound.

8. No sparrows are mammals.

9. All things that make me laugh these days are politicians self-destructing in public.

10. All places identical to Ottawa are capital cities of Canada.

11. All people who have a late-night snack are people who should brush their teeth.

12. No serial plagiarizers are students that are well-liked by their professors.

13. All victims of the defendant's con are very naïve and lonely people

14. Some outlaw motorcycle club members are people who get extremely high scores on the LSAT.

15. No people who eat everything on their plates are people who have to clean up the dinner dishes/ All people who eat everything on their plates are people who don't have to clean up the dinner dishes.

16. Some chefs in fancy restaurants are not people who like the food they prepare for their customers.

17. No people identical to Dr. Alward are very nice people/ All people identical to Dr. Alward are people who are not very nice.

18. Some people with poor personal hygiene and filthy clothing are not people who consider themselves to be undesirable romantic partners.

19. All people who use the anonymous tip line are people who don't want to get in trouble with the law/ No people who use the anonymous tip line are people who want to get in trouble with the law

20. All times egregious logical fallacies are committed are times Dr. Alward arrives to save the day.

Section 4.4

1. *Converse*: All stylish dressers are people wearing fedoras – not LE
   Obverse: No people wearing fedoras are unstylish dressers – LE
   Contrapositive: All unstylish dressers are people not wearing fedoras – LE

2. *Converse*: No scary monsters are things that go bump in the night – LE
   Obverse: All things that go bump in the night are unscary monsters – LE
   Contrapositive: No unscary monsters are things that don't go bump in the night – not LE

3. *Converse*: Some ballerinas are people with foul-smelling feet – LE
Obverse: Some people with foul-smelling feet are not non-ballerinas – LE
Contrapositive: Some non-ballerinas are people without foul-smelling feet – not LE

4. *Converse*: Some secrets are not things I'd rather not say – not LE
Obverse: Some things I'd rather not say are non-secrets – LE
Contrapositive: Some non-secrets are not things I'd rather say – LE

5. *Converse*: All things that end up in the landfill are things which require constant maintenance. – not LE
Obverse: No things which require constant maintenance are things that don't end up in the landfill. – LE
Contrapositive: All things that don't end up in the landfill are things which don't require constant maintenance – LE

6. *Converse*: No items cluttering up my spice rack are spices that need to be roasted prior to use. - LE
Obverse: All spices that need to be roasted prior to use are items not cluttering up my spice rack. – LE
Contrapositive: No items not cluttering up my spice rack are spices that don't need to be roasted prior to use – not LE

7. *Converse*: Some people who prefer remote teaching are crotchety old professors. – LE
Obverse: Some crotchety old professors are not people who don't prefer remote teaching. – LE
Contrapositive: Some people who don't prefer remote teaching are people who are not crotchety old professors. – not LE

8. *Converse*: Some items developed in secret research laboratories are not new-fangled flavors of gum. – not LE
Obverse: Some new-fangled flavors of gum are items not developed in secret research laboratories. – LE
Contrapositive: Some items not developed in secret research laboratories are not things that are not new-fangled flavors of gum. – LE

9. *Converse*: All people with little experience on the farm are people who think they can get my goat. – not LE
Obverse: No people who think they can get my goat are people with much experience on the farm. – LE
Contrapositive: All people with much experience on the farm are people who don't think they can get my goat. – LE

10. *Converse*: No stories permitted to be told in my living room are asinine tales of virtue and heroism. – LE

Obverse: <u>All</u> asinine tales of virtue and heroism <u>are</u> stories not permitted to be told in my living room. – LE
Contrapositive: <u>No</u> stories not permitted to be told in my living room <u>are</u> things that are not asinine tales of virtue and heroism. – not LE

11. *Converse*: <u>Some</u> things which pose the risk of uncomfortable dinner dates <u>are</u> strategies for getting acquaintances to help you move. – LE
Obverse: <u>Some</u> strategies for getting acquaintances to help you move <u>are not</u> things which don't pose the risk of uncomfortable dinner dates. – LE
Contrapositive: <u>Some</u> things which don't pose the risk of uncomfortable dinner dates <u>are</u> things that are not strategies for getting acquaintances to help you move. – not LE

12. *Converse*: <u>Some</u> people who reject conspiracy theories <u>are not</u> people whose views are resistant to contrary evidence. – not LE
Obverse: <u>Some</u> people whose views are resistant to contrary evidence <u>are</u> people who accept conspiracy theories. – LE
Contrapositive: <u>Some</u> people who accept conspiracy theories <u>are not</u> people whose views are not resistant to contrary evidence. – LE

Section 5.1

1. *Major term*: dogs with funny names
Minor term: lords of the household
Middle Term: beloved pets
Major Premise: All dogs with funny names are beloved pets
Minor Premise: Some lords of the household are beloved pets

2. *Major term*: people with 70's mustaches
Minor term: refugees from the 60's
Middle Term: friends of mine
Major Premise: No people with 70's mustaches are friends of mine
Minor Premise: Some friends of mine are refugees from the 60's

3. *Major term*: edible clothes items
Minor term: poor rain garments
Middle Term: things that dissolve in water
Major Premise: Some edible clothes items are things that dissolve in water
Minor Premise: All things that dissolve in water are poor rain garments

4. *Major term*: flying pink elephants
Minor term: memories of youth
Middle Term: fatigued-induced hallucinations
Major Premise: No flying pink elephants are fatigue-induced hallucinations

Minor Premise: All fatigued-induced hallucinations are memories of youth

5. *Major term*: compelling investment opportunities
   Minor term: illegal enterprises
   Middle Term: hare-brained schemes
   Major Premise: Some hare-brained schemes are compelling investment opportunities
   Minor Premise: All hare-brained schemes are illegal enterprises

6. *Mood*: OAO
   Figure: 1
   Valid/ invalid: invalid

7. *Mood*: AAA
   Figure: 1
   Valid/ invalid: valid

8. *Mood*: EAE
   Figure: 3
   Valid/ invalid: invalid

9. *Mood*: IAI
   Figure: 3
   Valid/ invalid: valid

10. *Mood*: EAE
    Figure: 2
    Valid/ invalid: valid

11. *Mood*: AII
    Figure: 4
    Valid/ invalid: invalid

12. *Mood*: EIO
    Figure: 3
    Valid/ invalid: valid

13. *Mood*: AAA
    Figure: 2
    Valid/ invalid: invalid

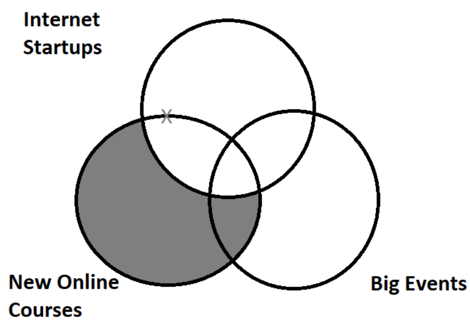
14. *Mood*: AOO
    Figure: 1

Valid/ invalid: invalid

15. *Mood*: AEE
Figure: 4
Valid/ invalid: valid

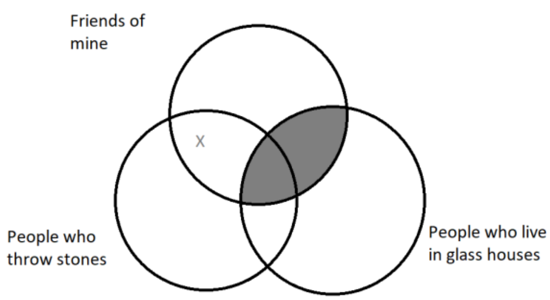
Section 5.2

1. *Diagram*:



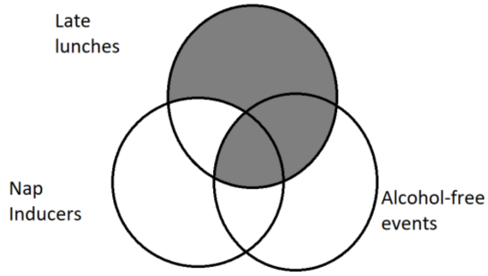
Conclusion type: O statement
Required features: x in 2 or 5 *Present?* No
Valid/ Invalid: Invalid

2. *Diagram*:



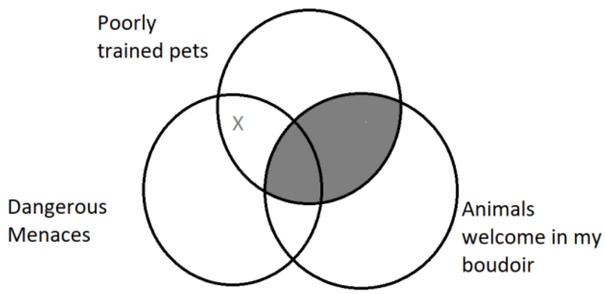
Conclusion type: E statement
Required features: shading in 3 and 6 *Present?* No
Valid/ Invalid: Invalid

3. *Diagram*:



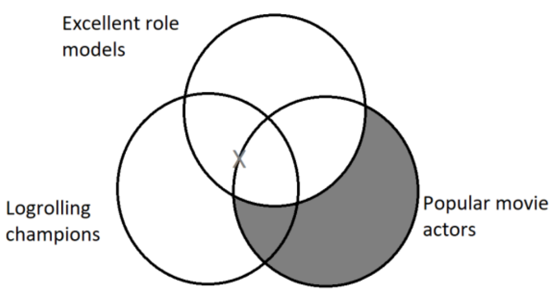
Conclusion type: E Statement
Required features: shading in 3 and 6 *Present?* No
Valid/ Invalid: invalid

4. *Diagram*:



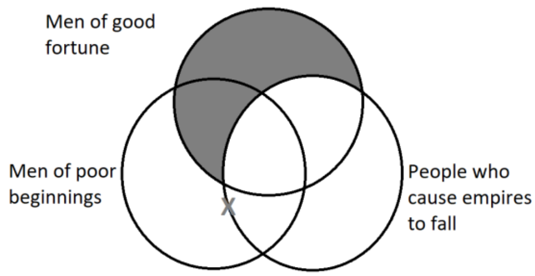
Conclusion type: O statement
Required features: x in 2 or 5 *Present?* Yes
Valid/ Invalid: valid

5. *Diagram*:



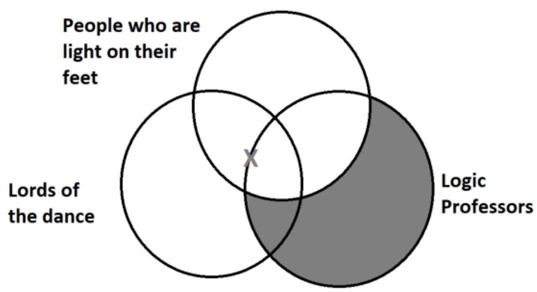
Conclusion type: I statement
Required features: x in 3 or 6 *Present?* No
Valid/ Invalid: Invalid

6. *Diagram*:



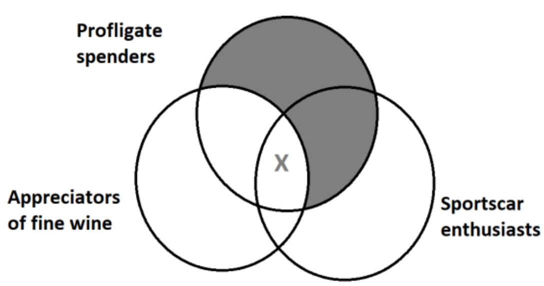
Conclusion type: O statement
Required features: x in 2 or 5 *Present?* No
Valid/ Invalid: invalid

7. *Diagram*:



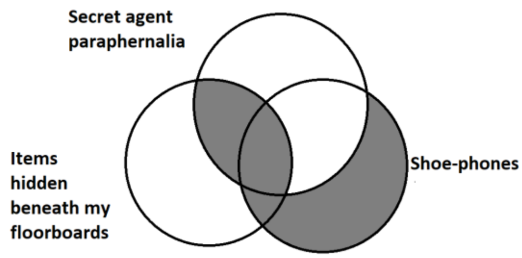
Conclusion type: E statement
Required features: shading in 3 and 6 *Present?* No
Valid/ Invalid: invalid

8. *Diagram*:



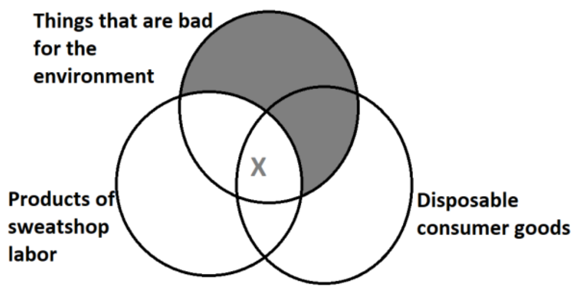
Conclusion type: I statement
Required features: x in 3 or 6 *Present?* Yes
Valid/ Invalid: valid

9. *Diagram*:



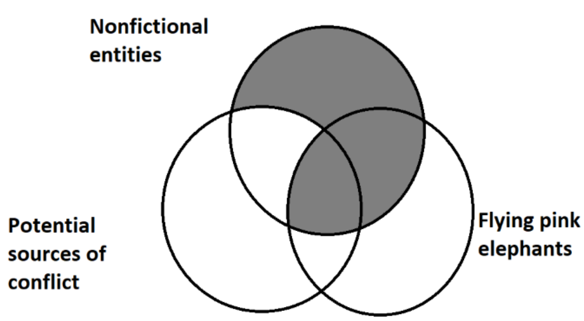
Conclusion type: E statement
Required features: shading in 3 and 6 *Present?* Yes
Valid/ Invalid: valid

10. *Diagram*:



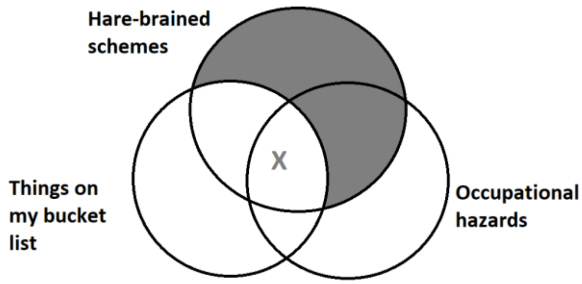
Conclusion type: I statement
Required features: x in 3 or 6 *Present?* yes
Valid/ Invalid: valid

11. *Diagram*:



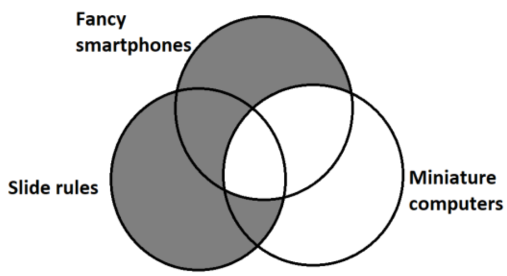
Conclusion type: I statement
Required features: x in 3 or 6 *Present?* No
Valid/ Invalid: invalid

12. *Diagram*:



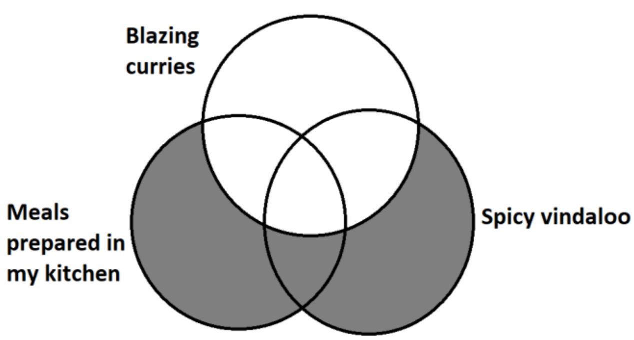
Conclusion type: I statement
Required features: x in 3 or 6 *Present?* yes
Valid/ Invalid: valid

13. *Diagram*:



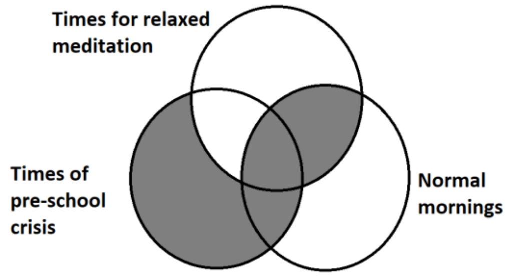
Conclusion type: A statement
Required features: shading in 2 and 5 *Present?* Yes
Valid/ Invalid: valid

14. *Diagram*:



Conclusion type: A statement
Required features: shading in 2 and 5 *Present?* No
Valid/ Invalid: Invalid

15. *Diagram*:



Conclusion type: E statement
Required features: shading in 3 and 6 *Present?* Yes
Valid/ Invalid: Valid

Section 5.3

1.	P1: Some O are not B ->	obversion ->	Some O are non-B
	P2: All non-P are O ->	no change ->	All non-P are O
	C: Some P are not B ->	contrapositive ->	Some non-B are not non-P
2.	P1: No non-S are G ->	no change ->	No non-S are G
	P2: Some G are E ->	no change ->	Some G are E
	C: No E are S ->	obversion ->	All E are non-S
3.	P1: No S are B ->	conv. + ob. ->	All B are non-S
	P2: All non-S are H ->	no change ->	All non-S are H
	C: No H are B ->	no change ->	No H are B
4.	P1: No W are D ->	no change ->	No W are D
	P2: Some W are N ->	no change ->	Some W are N
	C: Some N are not non-D ->	obversion ->	Some N are D
5.	P1: All non-B are I ->	no change ->	All non-B are I
	P2: Some I are V ->	no change ->	Some I are V
	C: Some V are B ->	obversion ->	Some V are not non-B
6.	P1: All non-M are C ->	no change ->	All non-M are C
	P2: Some P are not M ->	obversion ->	Some P are non-M
	C: Some P are not non-C ->	obversion ->	Some P are C
7.	P1: All S are non-H ->	obversion ->	No S are H
	P2: Some non-D are H ->	con. + Ob. ->	Some H are not D
	C: No D are S ->	no change ->	No D are S
8.	P1: Some L are A ->	no change ->	Some L are A

	P2: All non-L are D ->	contraposition ->	All non-D are L
	C: Some non-D are A ->	no change ->	Some non-D are A
9.	P1: All non-J are B ->	no change ->	All non-J are B
	P2: No B are non-F ->	obversion ->	All B are F
	C: No F are J ->	obversion ->	All F are non-J
10.	P1: Some E are V ->	obversion ->	Some E are non-V
	P2: All V are D ->	contraposition ->	All non-D are non-V
	C: Some non-D are non-E ->	obversion ->	Some non-D are not E
11.	P1: No non-C are non-H ->	obversion ->	All non-C are H
	P2: All H are F ->	obversion ->	No H are non-F
	C: Some non-F are C ->	obversion ->	Some non-F are not non-C
12.	P1: Some C are non-V ->	obversion ->	Some C are not V
	P2: All non-C are P ->	contraposition ->	All non-P are C
	C: Some non-P are V ->	no change ->	Some non-P are V
13.	P1: All non-M are C ->	no change ->	All non-M are C
	P2: All non-A are M ->	obversion ->	No non-A are non-M
	C: All A are non-C ->	contraposition ->	All C are non-A
14.	P1: All non-F are non-A ->	no change ->	All non-F are non-A
	P2: All non-P are A ->	contraposition ->	All non-A are P
	C: All P are F ->	obversion ->	No P are non-F
15.	P1: No non-L are non-W ->	no change ->	No non-L are non-W
	P2: All non-Y are W ->	contraposition ->	All non-W are Y
	C: No Y are L ->	obversion ->	All Y are non-L

Section 6.2

1. C is a necessary cause of the phenomenon – agreement

2. A is a necessary and sufficient cause – joint method

3. C is a sufficient cause – difference

4. D is a necessary and sufficient cause – joint method

5. E is a sufficient cause - difference

6. B is a necessary cause – agreement

7. E is a necessary cause – agreement

8. B is a necessary and sufficient cause – joint method

9. A is a sufficient cause – difference

10. A is a necessary and sufficient cause – joint method

11.(i)

Occurrence	*Drink Infected Water*	*Eat Infected Food*	*Breathe Infected Air*	*Infected Cuts*	**Zombie Plague**
Mary	Y	N	Y	Y	Y
Fred	N	N	Y	Y	Y
Jada	N	Y	Y	Y	Y
Rashid	Y	Y	Y	N	Y

(ii) Breathing infected air is (iii) a necessary cause of the zombie plague.

12. (i)

Occurrence	*Wine*	*Tablet*	*Snack*	*Homework*	**Insomnia**
Mary	Y	Y	Y	Y	Y
Jada	Y	N	Y	Y	N

(ii) using a tablet is a (iii) sufficient cause of insomnia

13. (i)

Occurrence	*Defective Transmitter*	*Defective Airlock*	*Defective Harness*	*Insulted HAL*	**Lost in Space**
Mary	Y	Y	Y	N	N
Fred	Y	N	Y	Y	Y
Jada	Y	Y	Y	Y	Y
Rashid	N	N	Y	N	N
Dora	N	N	Y	N	N
Diego	N	Y	N	Y	Y

(ii) insulting HAL is a (iii) necessary and sufficient cause of being lost in space.

14. (i)

Occurrence	*Generous Tip*	*Study Hard*	*Come to Class*	*Participate*	**90% grade**
Mary	Y	Y	Y	N	Y
Fred	Y	N	Y	N	Y
Jada	Y	Y	N	Y	Y
Rashid	Y	N	Y	Y	Y

(ii) leaving a generous tip is a (iii) necessary cause of getting a grade of 90% or higher

15. (i)

Occurrence	*No Updates*	*Spills*	*Downloads*	*Passwords*	**Crashes**
Mary's Laptop	Y	N	Y	Y	N
Jada's Laptop	Y	Y	Y	Y	Y

(ii) spilling things on a laptop is a (iii) sufficient cause of frequent laptop crashing

16. (i)

Occurrence	*Left Early for Airport*	*Checked in Online*	*Carry On Luggage*	*Valid ID*	**Make Flight**
Mary	Y	N	Y	N	Y
Fred	N	N	Y	Y	N
Jada	N	Y	Y	Y	N
Rashid	Y	Y	N	Y	Y
Dora	N	Y	N	Y	N
Diego	Y	N	Y	Y	Y

(ii) leaving early for the airport is a (iii) necessary and sufficient cause of making a flight

17. (i)

Occurrence	*King Pao Chicken*	*Big Mac*	*Lasagna*	*Fried Okra and Black-eyed Peas*	**Unpleasant Fridge Odor**
Mary	Y	Y	N	Y	Y
Fred	N	Y	Y	Y	Y
Jada	Y	Y	Y	N	Y
Rashid	Y	Y	N	N	Y

(ii) the leftover Big Mac is a (iii) necessary cause of unpleasant fridge odors

18. (i)

Occurrence	*Obedience Classes*	***Crate Training***	*Doggy Daycare*	*Human Socialization*	*Regular Walks*	*Chew Toys*	**Good with strangers**
Mary's Puppy	Y	**N**	Y	Y	Y	Y	N
Jada's Puppy	Y	**Y**	Y	Y	Y	Y	Y

(ii) crate training is a (iii) sufficient cause of being good with strangers

19. (i)

Occurrence	*Attend Practices*	*Neglect Schoolwork*	*Bribe Team Captain*	***Love Potion***	**Make Quidditch Team**
Mary	Y	N	Y	**Y**	Y
Fred	Y	Y	N	**Y**	Y
Jada	Y	N	Y	**N**	N
Rashid	Y	Y	Y	**Y**	Y
Dora	Y	N	N	**N**	N
Diego	Y	Y	N	**N**	N

(ii) give the captain a love potion is a (iii) necessary and sufficient cause of making the Slytherin House quidditch team

20. (i)

Occurrence	*Paid Admission*	*Spent $20 on Food*	*Made a Wish*	*Trapped on Ride*	*Did Good Deed*	*Set High Score*	**Free Admission Coupon**
Mary	Y	Y	N	N	Y	N	Y
Fred	Y	Y	Y	Y	N	N	Y
Jada	Y	Y	Y	Y	N	Y	Y
Rashid	Y	Y	Y	Y	N	Y	Y
Dora	Y	N	N	Y	N	Y	Y
Diego	Y	N	N	Y	N	Y	Y

(ii) paying for admission is a (iii) necessary cause of receiving a free admission coupon

21. (i)

Occurrence	*Cases the Joint*	*Mask and Gloves*	*Subdues Guards*	*Police Scanner*	*Vault Combination*	*Getaway Car*	**Successful robbery**
Mary's Heist	Y	Y	Y	Y	Y	Y	Y
Jada's Heist	Y	Y	Y	N	Y	Y	N

(ii) listening to the police scanner is a (iii) sufficient cause of a successful bank robbery

22. (i)

Occurrence	*Safety Gear*	*Pay Attention*	*Stay Hydrated*	*Practice at Home*	*Avoid Half-pipe*	*Easy Tricks*	**Avoid Injury**
Mary	Y	Y	Y	N	Y	N	Y
Fred	Y	Y	Y	Y	N	Y	N
Jada	Y	N	Y	Y	Y	Y	Y
Rashid	Y	N	N	Y	N	Y	N
Dora	Y	Y	Y	N	Y	Y	Y
Diego	Y	N	N	Y	N	N	N
Namid	N	N	Y	N	Y	Y	Y
Mikom	Y	Y	Y	N	N	N	N

(ii) avoiding the half-pipe is a (iii) necessary and sufficient cause of avoiding skateboarding injuries

23. (i)

Occurrence	*Monkey Paw*	*Magic Lamp*	*Evil Eye*	*Spell Cast*	*Drink Potion*	*Séance*	**Turned into Chicken**
Mary	Y	N	Y	Y	Y	Y	Y
Fred	Y	Y	N	N	Y	N	Y
Jada	Y	Y	Y	Y	Y	Y	Y
Rashid	Y	N	Y	Y	Y	Y	Y
Dora	Y	N	Y	Y	Y	N	Y
Diego	N	Y	N	Y	Y	N	Y

(ii) drinking a magic potion is a (iii) necessary cause of being transformed into a chicken

Section 7.1

1. ~B

2. S & ~C

3. ~C v S

4. (C & W) v ~R

5. (C & W) & ~R

6. ~(C v W) or ~C & ~W

7. ~(P & S) or ~P v ~S

8. ~P v ~S or ~(P & S)

9. ~C & ~W or ~(C v W)

10. ~(W v T) & P

11. ~W & (~T v P)

12. R -> (G v W)

13. (G & W) -> R

14. R -> (G v W)

15. S -> (O -> H)

16. (O -> H) -> S

17. (I -> C) -> (~C v ~F)

18. (C -> I) v (~C & ~F)

19. P <-> ~(I v M)

20. (I & M) <-> ~P

21. (P <-> ~I) -> ~M

22. ~[(H <-> ~S) & (E -> ~B)]

23. (S -> H) v ~(E v B)

24. [~V v ~(W & P)] -> (Y <-> ~P)

25. [V & (Y v W)] -> (~W <-> P)

26. A -> [S -> (~D v ~E)]

27. [~S -> (A & ~D)] -> (E <-> ~A)

28. [~(D v T) -> M] & [F -> (S v ~W)]

29. [~M <-> ~(~T & F)] & [~W <-> (S v D)]

30. [(M <-> P) <-> (P <-> ~S)] & [(S & P) v ~(M v E)]

Section 7.2

Section I

1.	1. A -> (B & C)	P
	2. D & A	P
	3. A	&E: 2
	4. B & C	->E: 1, 3
	5. C	&E: 4
	6. D	&E: 2
	7. C & D	&I: 5, 6

2.	1. (A v B) -> C	P
	2. B & D	P
	3. B	&E: 2
	4. A v B	vI: 3
	5. C	->E: 1, 4
	6. D	&E: 2
	7. C & D	&I: 5, 6

3.	1. ((A & B) & C) & D	P
	2. (A & B) & C	&E: 1
	3. A & B	&E: 2
	4. B	&E: 3
	5. B v (E <-> F)	vI: 4

4.	1. B	P
	2. B	R: 1

	3. B & B	&I: 1, 2
	4. (B & B) v F	vI: 3
	5. (B & B) v F	R: 4
	6. [(B & B) v F] & [(B & B) v F]	&I: 4,5
5.	1. (A & B) & (C & D)	P
	2. [(B v E) & (C v F)] -> G	P
	3. A & B	&E: 1
	4. A	&E: 3
	5. B	&E: 3
	7. B v E	vI: 5
	8. C & D	&E: 1
	9. C	&E: 8
	10. C v F	vI: 9
	11. (B v E) & (C v F)	&I: 7, 10
	12. G	->E: 2, 11
	13. G & A	&I: 4, 12
6.	1. (A v B) & (B -> C)	P
	2. (B -> C) -> (A -> C)	P
	3. A v B	&E: 1
	4. B -> C	&E: 1
	5. A -> C	->E: 2, 4
	6. (A -> C) & (B -> C)	&I: 4, 5
	7. C	vE: 3, 6
7.	1. A -> (B -> (C -> D))	P
	2. (A & B) & C	P
	3. A & B	&E: 2
	4. A	&E: 3
	5. B -> (C -> D)	->E: 1, 4
	6. B	&E: 3
	7. C -> D	->E: 5, 6
	8. C	&E: 2
	9. D	->E: 7, 8
	10. D v E	vI: 9
	11. (D v E) & A	&I: 4, 10
8.	1. (A -> B) & (C -> (B -> A))	P
	2. (C v D) & (D -> (B -> A))	P
	3. A -> B	&E: 1
	4. C -> (B -> A)	&E: 1
	5. D -> (B -> A)	&E: 2
	6. C v D	&E: 2
	7. (C -> (B -> A)) & (D -> (B -> A))	&I: 4, 5
	8. B -> A	vE: 6, 7
	9. (A -> B) & (B -> A)	&I: 3, 8
	10. A <-> B	<->I: 9

9.	1. A <-> B	P
	2. (A -> B) & (B -> A)	<->E: 1
	3. A -> B	&E: 2
	4. B -> A	&E: 2
	5. (B -> A) & (A -> B)	&I: 3, 4
	6. B <-> A	<->I: 5
10.	1. (A -> C) <-> B	P
	2. B & (C -> A)	P
	3. ((A -> C) -> B) & (B -> (A -> C))	<->E: 1
	4. B -> (A -> C)	&E: 3
	5. B	&E: 2
	6. A -> C	->E: 4, 5
	7. C -> A	&E: 2
	8. (A -> C) & (C -> A)	&I: 6, 7
	9. A <-> C	<->I: 8
11.	1. A -> (B & C)	P
	2. (B -> D) & A	P
	3. A	&E: 2
	4. B & C	->E: 1, 3
	5. B	&E: 4
	6. B -> D	&E: 2
	7. D	->E: 5, 6
	8. C	&E: 4
	9. C & D	&I: 7, 8
12.	1. A <-> B	P
	2. B <-> C	P
	3. A v C	P
	4. (A -> B) & (B -> A)	<->E: 1
	5. A -> B	&E: 4
	6. (B -> C) & (C -> B)	<->E: 2
	7. C -> B	&E: 6
	8. (A -> B) & (C -> B)	&I: 5, 7
	9. B	vE: 3, 8
13.	1. A -> (B -> (C -> D))	P
	2. B -> (A -> (D -> C))	P
	3. A & B	P
	4. A	&E: 3
	5. B -> (C -> D)	->E: 1, 4
	6. B	&E: 3
	7. C -> D	->E: 5, 6
	8. A -> (D -> C)	->E: 2, 6
	9. D -> C	->E: 4, 8
	10. (D -> C) & (C -> D)	&I: 7, 9

	11. D <-> C	<->I: 10

14.	1. A <-> B	P
	2. [(A -> B) -> (D -> C)] & [(B -> A) -> (C -> D)]	P
	3. (A -> B) & (B -> A)	<->E: 1
	4. A -> B	&E: 3
	5. B -> A	&E: 3
	6. (A -> B) -> (D -> C)	&E: 2
	7. (B -> A) -> (C -> D)	&E: 2
	8. D -> C	->E: 4, 6
	9. C -> D	->E: 5, 7
	10. (D -> C) & (C -> D)	&I: 8, 9
	11. D <-> C	<->I: 10

15.	1. A <-> (B -> C)	P
	2. A & (D v E)	P
	3. D -> (C -> B)	P
	4. A -> [E -> (C -> B)]	P
	5. (A -> (B -> C)) & ((B -> C) -> A)	<->E: 1
	6. A -> (B -> C)	&E: 5
	7. A	&E: 2
	8. B -> C	->E: 6, 7
	9. D v E	&E: 2
	10. E -> (C -> B)	->E: 4, 7
	11. [D -> (C -> B)] & [E -> (C -> B)]	&I: 3, 10
	12. C -> B	vE: 9, 11
	13. (B -> C) & (C -> B)	&I: 8, 12
	14. B <-> C	<->I: 13

Section II

1.	1. A -> B	P
	2. B -> C	P
	3. A	A
	4. B	->E: 1, 3
	5. C	->E: 2, 4
	6. A -> C	->I: 3-5

2.	1. A -> B	P
	2. ~B	P
	3. A	A
	4. B	->E: 1, 3
	5. B & ~B	&I: 2, 4
	6. ~A	~I: 3-5

3.	1. A -> (B & C)	P
	2. C -> A	P
	3. A	A

```
          4. B & C                              ->E: 1, 3
          5. C                                  &E: 4
      6. A -> C                                 ->I: 3-5
      7. (A -> C) & (C -> A)                    &I: 2, 6
      8. A <-> C                                <->I: 7

4.    1. (A v ~C) -> B                          P
      2. (C -> D) & ~B                          P
          3. ~C                                 A
          4. A v ~C                             vI: 3
          5. B                                  ->E: 1, 4
          6. ~B                                 &E: 2
          7. B & ~B                             &I: 5, 6
      8. C                                      ~E: 3-7
      9. C -> D                                 &E: 2
      10. D                                     ->E: 8, 9

5.    1. A -> (B & ~C)                          P
      2. B -> C                                 P
          3. A                                  A
          4. B & ~C                             ->E:
          5. B                                  &E: 4
          6. C                                  ->E: 2, 5
          7. ~C                                 &E: 4
          8. C & ~C                             &I: 6, 7
      9. ~A                                     ~I: 3-8

6.    1. B                                      P
          2. A                                  A
              3. C                              A
                  4. D                          A
                  5. B                          R: 1
              6. D -> B                         ->I: 4-5
          7. C -> (D -> B)                      ->I: 3-6
      8. A -> (C -> (D -> B))                   ->I: 2-7

7.    1. A -> ~B                                P
      2. ~B -> ~A                               P
      3. C -> A                                 P
          4. C                                  A
          5. A                                  ->E: 3, 4
          6. ~B                                 ->E: 1, 5
          7. ~A                                 ->E: 2, 6
          8. A & ~A                             &I: 5, 7
      9. ~C                                     ~I: 4-8

8.    1. A <-> B
      2. C -> (D -> B)
```

	3. D v A	
	4. C	
	5. D -> B	
	6. (A -> B) & (B -> A)	
	7. A -> B	
	8. (D -> B) & (A -> B)	
	9. B	
	10. C -> B	

9.	1. A -> ~B	P
	2. B	P
	3. ~A -> D	P
	4. C	A
	5. A	A
	6. ~B	->E: 1, 5
	7. B & ~B	&I: 2, 6
	8. ~A	~I: 5-7
	9. D	->E: 3, 8
	10. D v E	vI: 9
	11. C -> (D v E)	->I: 4-10

10.	1. ~A v B	P
	2. A	A
	3. ~A	A
	4. ~B	A
	5. A & ~A	&I: 2, 3
	6. B	~E: 4-5
	7. ~A -> B	->I: 3-6
	8. B	A
	9. B	R: 8
	10. B -> B	->I: 8-9
	11. (~A -> B) & (B -> B)	&I: 7, 10
	12. B	vE: 1, 11
	13. A -> B	->I: 2-12

11.	1. ~(A v B)	P
	2. A	A
	3. A v B	vI: 2
	4. (A v B) & ~(A v B)	&I: 1, 3
	5. ~A	~I: 2-4
	6. B	A
	7. A v B	vI: 6
	8. (A v B) & ~(A v B)	&I: 1, 7
	9. ~B	~I: 6-8
	10. ~A & ~B	&I: 5, 9

12.	1. A & B			P
		2. A		A
		3. B		&E: 1
	4. A -> B			->I: 2-3
		5. B		A
		6. A		&E: 1
	7. A -> B			->I: 5-6
	8. (A -> B) & (B -> A)			&I: 4, 7
	9. A <-> B			<->I: 8

13.	1. A v B			P
	2. ~B			P
		3. A		A
		4. A		R: 3
	5. A -> A			->I: 3-4
		6. B		A
			7. ~A	A
			8. B & ~B	&I: 2, 6
		9. A		~E: 7-8
	10. B -> A			->I: 6-9
	11. (A -> A) & (B -> A)			&I: 5, 10
	12. A			vE: 1, 11

14.	1. A v B		P
	2. A -> C		P
	3. B -> D		P
		4. A	A
		5. C	->E: 2, 4
		6. C v D	vI: 5
	7. A -> (C v D)		->I: 4-6
		8. B	A
		9. D	->E: 3, 8
		10. C v D	vI: 9
	12. B -> (C v D)		-> I: 8-10
	13. (A -> (C v D)) & (B -> (C v D))		&I: 7, 12
	14. C v D		vE: 1, 13

15.	1. A <-> B		P
	2. B <-> C		P
	3. (A -> B) & (B -> A)		<->E: 1
	4. (B -> C) & (C -> B))		<->E: 2
		5. A	A
		6. A -> B	&E: 3
		7. B	->E: 5, 6
		8. B -> C	&E: 4
		9. C	->E: 7, 8
	10. A -> C		->I: 5-9

	11. C	A
	12. C -> B	&E: 4
	13. B	->E: 11, 12
	14. B -> A	&E: 3
	15. A	->E: 13, 14
	16. C -> A	->I: 11-15
	17. (A -> C) & (C -> A)	&I: 10, 16
	18. A <-> C	<->I: 17

16.	1. A <-> B	P
	2. (C -> B) & (~C -> ~B)	P
	3. (A -> B) & (B -> A)	<->E: 1
	4. C	A
	5. C -> B	&E: 2
	6. B	->E: 4, 5
	7. B -> A	&E: 3
	8. A	->E: 6, 7
	9. C -> A	->I: 4-8
	10. A	A
	11. ~C	A
	12. ~C -> ~B	&E: 2
	13. ~B	->E: 11, 12
	14. A -> B	&E: 3
	15. B	->E: 10, 14
	16. B & ~B	&I: 13, 15
	17. C	~E: 11-16
	18. A -> C	->I: 10-17
	19. (C -> A) & (A -> C)	&I: 9, 18
20. C <-> A		<->I: 19

Section 8.2

1.

~A	->	(B	v	A)
F		T		F
T		T		F
T			T	
	T			

2.

~(A	->	~C)
F		T
F		F
	T	
F		

3.

(A	&	~B)	v	(~A	&	B)
F		T		F		T
F		F		T		T
	F				T	
			T			

4.

(C	<->	D)	<->	(D	&	C)
T		F		F		T
	F				F	
			T			

5.

~(A	&	~(B	v	C))
F		T		T
F			T	
F		F		
	F			
T				

6.

(A	<->	~D)	->	[(C	v	B)	&	~A]
F		F		T		T		F
F		T			T			T
	F						T	
			T					

Section 8.3

1. Logically True

A	(~A	&	A)	->	~(A	v	~A)
T	F	F		T	F	T	F
F	T	F		T	F	T	T

2. Logically Contingent

A	B	(A	<->	~B)	&	(~A	v	~B)
T	T		F	F	F	F	F	F
T	F		T	T	T	F	T	T
F	T		T	F	T	T	T	F
F	F		F	T	F	T	T	T

3. Logically Contingent

A	B	(A	->	~B)	<->	(~A	->	B)
T	T		F	F	F	F	T	
T	F		T	T	T	F	T	

F	T	T	F	T	T	T
F	F	T	T	F	T	F

4. Logically Contingent

A	**B**	**C**	**~A**	**&**	**(B**	**v**	**(C**	**<->**	**A))**
T	T	T	F	F		T		T	
T	T	F	F	F		T		F	
T	F	T	F	F		T		T	
T	F	F	F	F		F		F	
F	T	T	T	T		T		F	
F	T	F	T	T		T		T	
F	F	T	T	F		F		F	
F	F	F	T	T		T		T	

5. Logically True

A	**B**	**C**	**[~A**	**->**	**(B**	**->**	**~C)]**	**->**	**[(~A**	**&**	**B)**	**->**	**~C]**
T	T	T	F	T		F	F	T	F	F		T	F
T	T	F	F	T		T	T	T	F	F		T	T
T	F	T	F	T		T	F	T	F	F		T	F
T	F	F	F	T		T	T	T	F	F		T	T
F	T	T	T	F		F	F	T	T	T		F	F
F	T	F	T	T		T	T	T	T	T		T	T
F	F	T	T	T		T	F	T	T	F		T	F
F	F	F	T	T		T	T	T	T	F		T	T

6. Logically Contradictory, Logically Inconsistent

A	**B**	**(~A**	**v**	**B)**	**(A**	**&**	**~B)**
T	T	F	T			F	F
T	F	F	F			T	T
F	T	T	T			F	F
F	F	T	T			F	T

7. Logically Equivalent, Logically Consistent

A	**B**	**A**	**->**	**(B**	**->**	**~A)**	**(A**	**&**	**B)**	**->**	**~A**
T	T		F		F	F		T		F	F
T	F		T		T	F		F		T	F
F	T		T		T	T		F		T	T
F	F		T		T	T		F		T	T

8. Logically Inconsistent

A	**B**	**(~A**	**v**	**~A)**	**&**	**~B**	**~(B**	**->**	**~A)**
T	T	F	F	F	F	F	T	F	F
T	F	F	F	F	F	T	F	T	F
F	T	T	T	T	F	F	F	T	T
F	F	T	T	T	T	T	F	T	T

9. Logically Consistent

A	B	C	A	<->	~(B	v	C)	[A	->	~B]	&	[~A	->	C]
T	T	T		F	F	T			F	F	F	F	T	
T	T	F		F	F	T			F	F	F	F	T	
T	F	T		F	F	T			T	T	T	F	T	
T	F	F		T	T	F			T	T	T	F	T	
F	T	T		T	F	T			T	F	T	T	T	
F	T	F		T	F	T			T	F	F	T	F	
F	F	T		T	F	T			T	T	T	T	T	
F	F	F		F	T	F			T	T	F	T	F	

10. Logically Consistent

A	B	C	A	->	~(B	<->	C)	~A	v	(C	<->	B)
T	T	T		F	F	T		F	T		T	
T	T	F		T	T	F		F	F		F	
T	F	T		T	T	F		F	F		F	
T	F	F		F	F	T		F	T		T	
F	T	T		T	F	T		T	T		T	
F	T	F		T	T	F		T	T		F	
F	F	T		T	T	F		T	T		F	
F	F	F		T	F	T		T	T		T	

11. Valid

A	B	(A	v	B)	->	~B	/	A	&	~B	//	~B	v	A
T	T		T		F	F			F	F		F	T	
T	F		T		T	T			T	T		T	T	
F	T		T		F	F			F	F		F	F	
F	F		F		T	T			F	T		T	T	

12. Invalid

A	B	(A	v	~B)	<->	B	/	A	v	(~B	&	B)	//	B	->	~A
T	T		T	F	T				T		F				F	F
T	F		T	T	F				T		F				T	F
F	T		F	F	F				F		F				T	T
F	F		T	T	F				F		F				T	T

13. Invalid

A	B	(A	<->	B)	<->	A	/	~A	v	B	//	~(B	v	~B)
T	T		T		T			F	T			F	T	
T	F		F		F			F	F			F	T	
F	T		F		T			T	T			F	T	
F	F		T		F			T	T			F	T	

14. Invalid

A	B	C	A	->	(~B	->	A)	/	(A	<->	C)	&	~B//	~(B	<->	~C)	v	B
T	T	T		T	F	T				T		F	F	T	F	F	T	
T	T	F		T	F	T				F		F	F	F	T	T	T	
T	F	T		(T)	T	T				T		(T)	T	F	T	F	(F)	
T	F	F		T	T	T				F		F	T	T	F	T	T	
F	T	T		T	F	T				F		F	F	T	F	F	T	
F	T	F		T	F	T				T		F	F	F	T	T	T	
F	F	T		T	T	F				F		F	T	F	T	F	F	
F	F	F		(T)	T	F				T		(T)	T	F	F	T	(F)	

15. Valid

A	B	C	(~A	->	~C)	&	(B	->	~C)	/	(~A	v	B)	&	~B	//	~(B	v	C)
T	T	T	F	T	F	F		F	F		F	T		F	F		F	T	
T	T	F	F	T	T	T		T	T		F	T		F	F		F	T	
T	F	T	F	T	F	T		T	F		F	F		F	T		F	T	
T	F	F	F	T	T	T		T	T		F	F		F	T		T	F	
F	T	T	T	F	F	F		F	F		T	T		F	F		F	T	
F	T	F	T	T	T	T		T	T		T	T		F	F		F	T	
F	F	T	T	F	F	F		T	F		T	T		T	T		F	T	
F	F	F	T	T	T	(T)		T	T		T	T		(T)	T		(T)	F	